

Model of phase distribution of hydrophobic organic chemicals in cyclodextrin–water–air–solid sorbent systems as a function of salinity, temperature, and the presence of multiple CDs

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Abstract Environmental and other applications of cyclodextrins (CD) often require usage of high concentration aqueous solutions of derivatized CDs. In an effort to reduce the costs, these studies also typically use technical grades where the purity of the CD solution and the degree of substitution has not been reported. Further, this grade of CD often included high levels of salt and it is commonly applied in high salinity systems. The mathematical models for water and air partitioning coefficients of hydrophobic organic chemicals (HOC) with CDs that have been used in these studies under-estimate the level of HOC within CDs. This is because those models (1) do not take into account that high concentrations of CDs result in significantly lower levels of water in solution and (2) they do not account for the reduction in HOC aqueous solubility due to the presence of salt. Further, because they have poor knowledge of the CD molar concentration in their solutions, it is difficult to draw comparisons between studies. Herein is developed a mathematical model where cyclodextrin is treated as a separate phase whose relative volume is calculated from its apparent molar volume in solution and the CD concentration of the solution. The model also accounts for the affects of temperature and the presence of salt in solution through inclusion of modified versions of the Van't Hoff and Setschenow equations. With these capabilities, additional equations have been developed for

calculating HOC phase distribution in air–water–CD–solid sorbent systems for a single HOC and between water and CD for a system containing multiple HOCs as well as multiple types of cyclodextrin.

Keywords Cyclodextrin · Hydrophobic organic chemicals · Henry's Law constants · Molar volume · Model · Freundlich isotherm · NAPL

Introduction

Cyclodextrins (CD) are macro-ring molecules composed of glucopyranose units that have a variety of uses as carriers of hydrophobic organic chemicals (HOCs), chromatographic separating agents, and as viscosity additives [1]. The torodially shaped molecules have a polar exterior and a nonpolar interior cavity where HOCs of an appropriate size and shape can form inclusion complexes. Native CDs are non-toxic naturally occurring compounds, but their negligible aqueous solubility in comparison with synthetic CD derivatives has limited their application to groundwater remediation [2, 3]. There are over 1500 synthetic CD derivatives [1] among which 2-hydroxypropyl- β -cyclodextrin (HP- β -CD) is the most commonly used due to its relatively low cost and HOC solubility enhancement characteristics. Since 1993 [4], it has been systematically evaluated as an agent for remediation of hazardous waste sites. That CD and other derivatives were initially investigated for their ability to increase desorption and dissolution of HOCs and has evolved to evaluation of CD derivatives for remediation of mixed metal and HOC sites [5], increasing biodegradation of HOCs [6] and as a mechanism for increasing the efficiency of oxidants and their applications [7].

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Compared to other fields of chemistry and environmental science, publications that employed high concentrations of modified CDs such as those typically used in environmental restoration studies ($>100 \text{ g L}^{-1}$) [3, 8–10] have rarely drawn comparisons with their results and those of other studies. The primary reasons for this arise from the inherent variability in the purity the CDs and the average degree of substitution of the modified CDs that those studies used and the poor of reporting of this information when it is known. This fact is not completely the fault of the authors. In the past, CD manufacturers often did not provide this information because of variation within the quality of their CD production and due to cost concerns the choice by environmental studies to use less pure technical-grade CDs rather than high-purity grades that often used in pharmacology studies. As an example, modified CDs used in lab studies range in purity from 85 to 99 % and in case of HP- β -CD the average degree of substitution range from 0.9 to 5.6 and higher. Thus, two studies both using the same mass per volume concentration of 100 g L^{-1} HP- β -CD could differ in their molar concentration with values ranging from 55 and 82 mM.

Because knowledge of the purity, degrees of substitution, and level of salts within the CD powder or liquid as it was received was lacking, the equations being used in environmental CD studies relating guest interactions with CDs did not take into account the influence of salt impurities and used concentration units of mass of CD powder per volume of solution [3, 4, 8–11] rather than molar or molal units which make guest host interactions far simpler to explain. Further, the equations that were used in these studies were largely borrowed from the existing literature where it was most commonly used to describe the interactions of potential guest compounds with native CDs that are present in solution at comparatively dilute concentrations [12]. However, CD suppliers have improved the precision of their manufacturing methods and are now applying recently developed techniques for compositional analysis of modified CD. As such, conditions now exist for new models to be created which meet the needs of environmental and other high CD concentration applications to accurately assess guest–host interactions. Requisite in the design of these models is the capability of providing precise relationships for calculating HOC distribution in environmental systems where multiple phases such as air, water, and solid sorbents are present and which rightfully treat CD as a distinct separate phase due to its high concentration. Further, these models would need to be sufficiently robust to account for the influences of temperature and the presence of salts, multiple HOCs, and multiple types of cyclodextrin which affect guest–host HOC CD interactions in environmental systems.

The goals of this paper are: (1) to develop a mathematical model capable of accurately predicting the distribution of a single component HOC between air, water, cyclodextrin, and

solid sorbent phases as a function of the concentration of a single salt and temperature and (2) develop equations to describe phase distribution between water and a single cyclodextrin for mixtures of HOCs and the distribution of a single HOC between water and multiple types of cyclodextrin.

Mathematical models of HOC distribution in air–water–cyclodextrin systems

Because applications of cyclodextrin for environmental restoration typically employ highly concentrated solutions, a model is needed that does not treat CD as non-volume contributing pseudo-phase within water [11, 13–16]. To have CD as a distinct separate phase then it needs to be included in a mass balance equation beside other environmental phases such as air and water. The total mass of single component HOC in a system consisting of air, water, and CD phases can be found using the mass balance and volume balance Eqs. 1 and 2:

$$M_{\text{AT}} = C_{\text{A1}}V_1 + C_{\text{A2}}V_2 + C_{\text{A3}}V_3 \quad (1)$$

$$V_{\text{t}} = V_1 + V_2 + V_3 \quad (2)$$

where the subscript A is the designation for the HOC and the numbers 1, 2, and 3 are the respective designations for the air, water, and CD phases. M_{AT} is the total mass of the HOC in the system (moles_{A}), the various C terms are concentrations ($\text{moles}_{\text{A}} \text{ L}^{-1}$), and the V terms are the volumes of the phases (L). Because the purity of the CD powder is typically given in mass terms rather than volume of the CD phase, V_3 , is the net volume of CD rather than the powder and is calculated from the measured masses of the solution components, the purity and molar mass of the CD powder, and from partial molar volumes. Water is assumed to be completely held within V_2 because the amount within the cavities of CD is negligible. It has been shown in X-ray studies of aqueous CD solutions that water molecules can be present within the cyclodextrin cavity where they are theorized to form hydrogen bonds with the walls of the torus [17]. However, the number of water molecules within individual cyclodextrin molecules has been found to be only 5 to 6 for α -CD and 6 to 7 for β -CD [17]. Using a median value of 6.5 water molecules per HP- β -CD molecule, then in a 75 mM aqueous solution of this cyclic sugar ($\sim 100 \text{ g L}^{-1}$ solution of HP- β -CD with a degree of substitution of 2.7) the fraction of water molecules that are within the cavity would be approximately 0.9 % of the total in solution and less when partially displaced by inclusion of guest HOCs. Thus, it can be neglected. Because most CDs, including HP- β -CD, are also nonelectrolytes [17], they do not produce large hydration spheres where HOC levels would likely be reduced.

Therefore, these CDs should have a negligible change on the effective volume of water in that distinct phase available to the HOC and the presence of cyclodextrins should not greatly alter the relative density of water outside of the CD molecules (V_2). This is a fundamentally different affect than dissolved salts have on the aqueous solubility of HOCs. Due to their charged nature, salt ions create hydration spheres which effectively reduce the net volume of water available to accommodate and thus provide for the solubility of HOCs in aqueous solutions.

In a three phase air/water/CD system, the relative distribution of HOCs between air and water has commonly been assumed to follow the standard Henry's Law equation [13–16]:

$$K_{A12} = \frac{C_{A1}}{C_{A2}} \quad (3)$$

where K_{A12} is the air/water partitioning coefficient which is more commonly known as the standard Henry's Law Constant (HLC) (units of $L_{\text{water}} L_{\text{air}}^{-1}$). In these same publications, the relative HOC distribution between air and CD and between water and CD are also thought to have similar linear relationships:

$$K_{A13} = \frac{C_{A1}}{C_{A3}} \quad (4)$$

$$K_{A23} = \frac{C_{A2}}{C_{A3}} \quad (5)$$

where K_{A13} is the air/CD partitioning coefficient ($L_{\text{CD}} L_{\text{air}}^{-1}$) and K_{A23} is the water/CD partitioning coefficient ($L_{\text{CD}} L_{\text{water}}^{-1}$). The high aqueous solubility of HP- β -CD ($>800 \text{ g L}^{-1}$) and the relatively large molecular mass of the molecule likely greatly limit its vapor pressure and presence in the gas phase to level sufficiently low enough to ignore. As Lantz et al. [14] showed, the three partitioning coefficients can be related to each other through:

$$K_{A23} = \frac{K_{A13}}{K_{A12}} \quad (6)$$

For air in contact with CD solutions, the apparent HLC is defined in the terms of this model as:

$$K'_{A1\text{liq}} = \frac{C_{A1}}{C_{A\text{liq}}} = \frac{C_{A1} V_{\text{liq}}}{C_{A2} V_2 + C_{A3} V_3} \quad (7)$$

where $K'_{A1\text{liq}}$ is the apparent HLC (units of $L_{\text{liquid}} L_{\text{air}}^{-1}$) and the total volume of liquid, V_{liq} , equals the volume of the water and CD phases. The apparent HLC can be related to the other partitioning coefficients through:

$$K'_{A1\text{liq}} = \frac{C_{A1} V_{\text{liq}}}{\frac{C_{A1}}{K_{A12}} V_2 + \frac{C_{A1}}{K_{A13}} V_3} = \frac{K_{A12} V_{\text{liq}}}{V_2 + \frac{V_3}{K_{A23}}} \quad (8)$$

For solutions absent the presence of cyclodextrin (i.e., $V_3 = 0$ and $V_2 = V_{\text{liq}}$), this equation reduces to the

apparent HLC, K'_{A12} , equating to the actual HLC, K_{A12} . With this relationship for apparent HLC, the mass balance equation Eq. 1 can now be written as a two phase equation:

$$M_{\text{AT}} = C_{A1} V_1 + \frac{C_{A1}}{K'_{A1\text{liq}}} V_{\text{liq}} \quad (9)$$

The fact that cyclodextrin commonly represents a significant fraction of the liquid volume when it is employed as an agent for environmental restoration can be accounted for with knowledge of the molar concentration of this solution component and the apparent or partial molar volume of cyclodextrin and a relationship between the V_{liq} , V_2 , and V_3 for pure solutions can be found.

$$\frac{V_2}{V_{\text{liq}}} = 1 - V_{\text{mCD}} C_{\text{CD}} \quad (10)$$

$$\frac{V_3}{V_{\text{liq}}} = V_{\text{mCD}} C_{\text{CD}} = \phi_{\text{CD}} \quad (11)$$

where V_{mCD} is the apparent molar volume of one mole of the subject CD in solution, C_{CD} is the concentration of cyclodextrin in molar units, and ϕ_{CD} equals the volumetric fraction of the liquid accounted for by cyclodextrin. Incorporating Eqs. 10 and 11 into 8 results in a relationship between the apparent and actual HLCs, cyclodextrin concentration and the water/CD partitioning coefficient:

$$K'_{A1\text{liq}} = \frac{K_{A12}}{1 - V_{\text{mCD}} C_{\text{CD}} + \frac{V_{\text{mCD}} C_{\text{CD}}}{K_{A23}}} \quad (12)$$

Extending from the molar volume of CD, the fraction of the potential host molecules occupied with a guest HOC, f_{ACD} , is given by Eq. 13:

$$f_{\text{ACD}} = C_{A3} V_{\text{mCD}} = \frac{C_{A2}}{K_{A23}} V_{\text{mCD}} \quad (13)$$

In a single component HOC saturated aqueous solution, the concentration of the HOC in the water phase, C_{A2} , equals equilibrium aqueous solubility and f_{ACD} equals the maximum fraction of CD molecules occupied under equilibrium conditions.

Using measurements obtained with a densitometer, Zielenkiewicz et al. [18] reported a relationship for the apparent molar volume of CDs in solution with the concentration of the cyclic sugars in units of molality. Their relationship can be adapted to the molarity units and the choice of water as the solvent in the model developed here:

$$V_{\text{mCD}} = m_{\text{CD}}/\rho_{\text{liq}} + 10^3(\rho_{\text{liq}} - \rho_2)/(C_{\text{CD}}^* \omega_2 \rho_{\text{liq}} \rho_2) \quad (14)$$

where m_{CD} is the molar mass of the subject CD, C_{CD}^* is the concentration of CD in solution in molarity units, ω_2 is the mass of water per volume of liquid, ρ_{liq} is the density of the

liquid, and ρ_2 is the density of the solvent which in this case is water. While Blanford et al. [3] measured the density of aqueous solutions of HP- β -CD at ambient room temperature, they did not analyze density as a function of temperature. But more importantly, the inherent variability in the purity and degree of substitution of CD derivatives may require that density values be subject specific. Riberio et al. [19] reported density measurements of aqueous solutions of unmodified γ -cyclodextrin at two temperatures (298.15 and 310.15° K) and a range of CD concentrations (0.002–0.010 mol L⁻¹). For the conditions of their study, they projected a relationship between the solution density and CD concentration in molar units to be slightly non-linear and well approximated by the below equation:

$$\rho_{\text{liq}} = \rho_2(1 + b_1 C_{\text{CD}} + b_2 C_{\text{CD}}^2) \quad (15)$$

where b_1 and b_2 are calibrated coefficients. While they did find that the 10 °C temperature increase reduced the density of solution by an average of 0.4 %, they did not include a temperature dependence in their equation. In the case of the solvent water, equations for calculating the density of water as function of temperature and salinity are readily available [20] and Eqs. 14 and 15 can be further refined.

Under the conditions of their study, Zielenkiewicz et al. [18] found for an aqueous solution of uncomplexed HP- β -CD with a degree of substitution of 0.6 at a concentration of 4×10^{-2} mol kg⁻¹ and a density of water of 997.04 g L⁻¹ that V_{mCD} for HP- β -CD equaled 0.861 L mol⁻¹. In addition, their methods permitted calculation of the volume of their subject HP- β -CD when complexed with *p*-aminobenzoic acid which resulted in a value of 0.879 ± 0.0008 L mol⁻¹. As with many HOCs, their results show HOCs may only be partially encapsulated within the cavity of cyclodextrins.

Influence of salt on HOC distribution in CD liquids

The effect of salt reducing the aqueous solubility of organic chemicals has long been known and has historically been defined by the Setschenow salting out equation [21]. It is normally written as a logarithmic formula, but to make it more readily employed in this derivation it is shown in the form of a natural log:

$$\ln\left(\frac{C_{\text{Ao}}}{C_{\text{Asalt}}}\right) = k_s C_s \quad (16)$$

where C_{Ao} is the saturated aqueous solubility of the HOC, C_{Asalt} is the solubility limit of the HOC as a function of salt concentration, k_s is the salting out constant, and C_s is the ionic strength of the particular salt solution. In the literature, k_s and C_s appear in both molar and molal units. It should be noted that k_s values have repeatedly been found to be unique

to the particular elemental composition of the salt and the specific HOC and salting out coefficients for slightly polar molecules such as toluene are non-additive [22]. The ionic strength for the salting out equation is normally calculated from the Lewis and Randall's equation:

$$C_s = \frac{1}{2} \sum C_i Z_i^2 \quad (17)$$

where C_i is the molar or molal concentration of ionic species i and Z_i is the charge of the species. For salt solutions, C_s equals the molar or molal concentration of total salt, but because the choice affects the value of k_s the units of C_s need to be clearly stated. Several groups [23–25] have adapted the Setschenow salting out equation to Henry's coefficients through multiplying by the saturated vapor pressure.

$$\ln\left(\frac{C_{\text{A2}}}{C_{\text{A2salt}}}\right) = \ln\left(\frac{K_{\text{A12salt}}}{K_{\text{A12DI}}}\right) = k_s C_s \quad (18)$$

where K_{A12salt} is the apparent HLC for an air/salt water system and K_{A12DI} is the HLC for pure deionized water which is the same as K_{A12} . Reworking equation 18 yields:

$$K_{\text{A12salt}} = K_{\text{A12}} e^{k_s C_s} \quad (19)$$

Literature values on salting out coefficients are inherently variable where some have observed a temperature sensitivity for k_s [26] while others have found for their systems it is functionally insensitive temperature [27, 28]. Because this issue is potentially in doubt, the salting out coefficient is ascribed in this model as being temperature sensitive. Because the interior of the cyclodextrin cavity is non-polar, it is unlikely that any meaningful levels of salt ions will enter the CD torus and thus should remain within the water phase. As a result, the air/CD partitioning coefficient, K_{A13} , should not be affected by the presence of salt. If the assumption that the vapor pressure above a HOC saturated saline solution is the same as that above a pure water made by Peng and Wan [23], Falabella et al. [25], and Falabella and Teja [27] is accurate, then the HOC concentration within the CD cavities, C_{A3} , under HOC saturated conditions should be the same for both saline and pure water CD solutions. Therefore:

$$\frac{K_{\text{A23}}}{K_{\text{A23salt}}} = \frac{\frac{C_{\text{A2}}}{C_{\text{A3}}}}{\frac{C_{\text{A2salt}}}{C_{\text{A3}}}} = e^{k_s(T) C_s} \quad (20)$$

Thus, modifying the apparent HLC for the presence of salt would give:

$$K'_{\text{A1liqsalt}} = \frac{K_{\text{A12}} e^{k_s(T) C_s}}{1 - V_{\text{mCD}} C_{\text{CD}} + \frac{V_{\text{mCD}} C_{\text{CD}} e^{k_s(T) C_s}}{K_{\text{A23}}}} \quad (21)$$

Because K'_{Aliq} , K'_{Aliqsalt} , K_{A12} , V_{mCD} , C_s , $k_s(T)$, and C_{CD} can be measured Eq. 12 can be rearranged to provide an

equation to determine the CD/water partitioning coefficient (K_{A23}).

$$K_{A23} = \frac{V_{mCD}C_{CD}}{\frac{K_{A12}}{K_{A1liq}} - 1 + V_{mCD}C_{CD}} \quad (22)$$

And the salt water/CD partitioning coefficient, $K_{A23salt}$, can be found to be equal to:

$$K_{A23salt} = \frac{V_{mCD}C_{CD}}{\frac{K_{A12}e^{k_s(T)C_s}}{K_{A1liqsalt}} - 1 + V_{mCD}C_{CD}} \quad (23)$$

The temperature dependence of the above partitioning coefficients can be found with van't Hoff equation [29]:

$$K = e^{\left(\frac{E_1}{T} + E_2\right)} \quad (24)$$

Modifying the apparent HLC equation to include this basic temperature dependence gives:

$$K'_{A1liqsalt} = \frac{\exp\left(\frac{E_{1A12}}{T} + E_{2A12} + k_s(T)C_s\right)}{1 - V_{mCD}C_{CD} \left(1 + \exp\left(k_s(T)C_s - \frac{E_{1A23}}{T} - E_{2A23}\right)\right)} \quad (25)$$

HOC mass fraction by phase

Determination of the equilibrium phase distribution of HOCs between air, water, CD, and sorbent phases provides information on the potential performance of CDs in an environmental application and enables optimization of the design of CD delivery and treatment systems. For ternary phase diagrams, the below equations give the HOC mass fraction in the air, water, and CD phases. First the mass balance equation:

$$M_{AT} = C_{A1}V_1 + \frac{C_{A1}}{K_{A12}e^{k_sC_s}}V_{liq}(1 - V_{mCD}C_{CD}) + \frac{C_{A1}}{K_{A12}K_{A23}}V_{liq}V_{mCD}C_{CD} \quad (26)$$

Equation 26 can be reorganized by dividing through by the air concentration and the air volume and substituting φ as the liquid to gas volume ratio.

$$\frac{M_{AT}}{C_{A1}V_1} = 1 + K_{A12}e^{k_sC_s}\varphi(1 - V_{mCD}C_{CD}) + \frac{\varphi V_{mCD}C_{CD}}{K_{A12}K_{A23}} \quad (27)$$

From here the fractions of the HOC mass in each phase can be found by normalizing each of the three terms on the right hand side of Eq. 27 by the summation of that side. Thus, the fraction in the air phase becomes:

$$\frac{1}{1 + K_{A12}e^{k_sC_s}\varphi(1 - V_{mCD}C_{CD}) + \frac{\varphi V_{mCD}C_{CD}}{K_{A12}K_{A23}}} \quad (28)$$

Fraction in water:

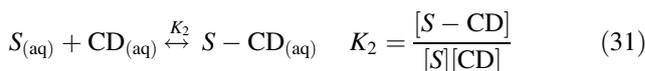
$$\frac{K_{A12}e^{k_sC_s}\varphi(1 - V_{mCD}C_{CD})}{1 + K_{A12}e^{k_sC_s}\varphi(1 - V_{mCD}C_{CD}) + \frac{\varphi V_{mCD}C_{CD}}{K_{A12}K_{A23}}} \quad (29)$$

Fraction in CD:

$$\frac{\frac{\varphi V_{mCD}C_{CD}}{K_{A12}K_{A23}}}{1 + K_{A12}e^{k_sC_s}\varphi(1 - V_{mCD}C_{CD}) + \frac{\varphi V_{mCD}C_{CD}}{K_{A12}K_{A23}}} \quad (30)$$

Relationship of K_{A23} with existing CD reaction models

In the literature, the interaction of cyclodextrin with a HOC in water has widely been described by reaction models [1, 4, 30, 31]. Equation 31 gives the most widely used model for a 1:1 complexation between CD and a HOC where $S_{(aq)}$ is the aqueous concentration of the HOC outside of the CD cavity (uncomplexed), $CD_{(aq)}$ is the concentration of uncomplexed CD in solution, $S-CD_{(aq)}$ is the concentration of the HOC-CD complex in solution and K_2 is referred to as the association binding constant or the stability constant.



While Eq. 31 is usually written with brackets which would normally imply chemical activity, in practice concentration units of mass S, CD, or S-CD per total liquid volume are used instead of activities. As stated earlier, Rekharsky et al. [17] reported that because most CDs are nonelectrolytes their activities will be close to one, but they stated that applies to low CD concentration solutions. Using activities would permit the equation to be applied to relatively high CD concentration solutions where the mass of water in solution is greatly reduced due to the level of CD (e.g. 100 g L⁻¹ CD solution has a reduction of ~7 % in the mass of water over a 0 g L⁻¹ solution). However, in practice the activity coefficients of constituents are assumed to be equal to one and in effect Eq. 31 uses concentrations. As a result, this model assumes the volume of water is equal the volume of solution. This is potentially misleading in the application of Wang and Brusseau [4] CD reaction equation by McCray et al. [11] where they created HOC saturated solutions to determine water-CD distribution coefficients. In their work, they assumed that the mass of uncomplexed HOC in solution equals literature values for aqueous solubility multiplied by the total solution volume rather than the volume of solution after deducting the portion of the total volume accounted for by the added cyclodextrin. Thus, their water-CD partitioning coefficients underestimate the proportion of HOC molecules within cyclodextrin cavities.

Previous efforts by Kashiyama and Boving [13], Lantz et al. [14], and Szaniszló et al. [15] to determine air/CD solution phase partitioning coefficients by headspace GC also made an effort to relate those coefficients to K_2 . Both Kashiyama and Boving [13] and Szaniszló et al. [15] designed their experiments with low levels of HOC so that they could assume that the concentration of complexed CD, [G-CD], is far less than the uncomplexed CD concentration, [CD], and thus the uncomplexed CD concentration equals the total CD concentration which is known from preparation. This design limits the applicability of their equation to low HOC concentration solutions. Szaniszló et al. [15] did not report the values of the Henry's Law constants they used in their analysis and they report a range of total guest mass in their preparations of their samples rather than an exact value, but attempts to produce a similar level of difference between uncomplexed and complexed CD using their methods were unsuccessful which shows the limitation of this method. While the Lantz et al. [14] model did have separate volumes for water and CD in the formulations of their partitioning coefficients, their equation relating K_2 and to their water/CD partitioning coefficient went against their own model's design by again assuming that the volume of water was equal to the volume of solution.

Using the nomenclature of this paper, below are the equivalent equations for reaction a model.

$$[S] = C_{A2} \frac{V_2}{V_{\text{liq}}} \quad [CD] = C_{CD} - C_{A3} \frac{V_3}{V_{\text{liq}}} \quad (32)$$

$$[S - CD] = C_{A3} \frac{V_3}{V_{\text{liq}}}$$

Thus:

$$K_2 = \frac{\left(\frac{C_{A3}V_3}{V_{\text{liq}}}\right)}{\left(\frac{C_{A2}V_2}{V_{\text{liq}}}\right)\left(C_{CD} - \frac{C_{A3}V_3}{V_{\text{liq}}}\right)} = \frac{V_{\text{mCD}}C_{CD}}{K_{A23}(1 - V_{\text{mCD}}C_{CD})(C_{CD} - C_{A3}V_{\text{mCD}}C_{CD})} \quad (33)$$

After reorganizing:

$$K_2 = \frac{V_{\text{mCD}}C_{CD}}{K_{A23}C_{CD}(1 - V_{\text{mCD}}C_{CD})(1 - C_{A3}V_{\text{mCD}})} = \frac{\phi_{\text{cd}}}{K_{A23}C_{CD}(1 - \phi_{\text{cd}})(1 - f_{\text{ACD}})} \quad (34)$$

where f_{ACD} again equals $C_{A3}V_{\text{mCD}}$, the fraction of CD molecules occupied with a HOC, and ϕ_{CD} again equals $V_{\text{mCD}}C_{CD}$, the fraction of the total solution volume accounted for by cyclodextrin.

The value of f_{ACD} is often high in saturated solutions of high aqueous solubility HOCs. (e.g. using the experimental results of McCray et al. 2000 and correcting for CD volume, in a TCE saturated 100 g L⁻¹ HP- β -CD solution with an assumed DS of 4.9 results in 71 % of the CD molecules

are occupied with this HOC). However, in CD systems with low HOC concentrations or in HOC saturated CD systems where the HOC has a low aqueous solubility then $f_{\text{ACD}} \ll 1$ and the above equation can be reduced to:

$$K_2 \cong \frac{1}{K_{A23}\left(\frac{1}{V_{\text{mCD}}} - C_{\text{CD}}\right)} \quad (35)$$

Solubility enhancement factor

McCray et al. [11] applied the concept of the solubility enhancement factor to define the degree to which solubility of an HOC is improved by an aqueous CD solution over that of pure water. Their model was based on the equation in Wang and Brusseau [4] in which the water/CD partitioning coefficient where the concentration of complexed cyclodextrin molecules are low.

$$S_A = S_W(1 + K_{\text{CW}}C_{\text{CD}}) = S_W E \quad (36)$$

where S_A equals the apparent solubility in aqueous CD solutions, S_W is the solubility in water, K_{CW} is the partitioning coefficient between water and CD, and E is the solubility enhancement factor.

While the methods of McCray et al. [11] show the effective increase in apparent solubility, they provide misleading information on the water/CD partitioning coefficient and their model can be misinterpreted where the fraction of HOC mass in water equals 1/E and fraction in CD is equal to (E-1)/E. Despite this, the idea of a solubility enhancement factor is useful especially when proposing cyclodextrins as a remedial alternative.

In the nomenclature of this paper, the enhancement factor can be determined from assuming the lack of an air phase and normalizing the remaining terms in the original mass balance equation by the volume of the liquid.

$$\frac{M_{\text{AT}}}{V_{\text{liq}}} = C_{A2} \left(1 + \left(\frac{V_{\text{mCD}}}{K_{A23}} - V_{\text{mCD}}\right)C_{\text{CD}}\right) \quad (37)$$

The left-hand side of Eq. 37 equals S_A , the apparent solubility in CD solutions. Since in a HOC saturated CD solution the HOC concentration in the water portion equals the aqueous solubility, C_{A2} equals S_W . Thereby, the remaining terms fulfill the notion of a solubility enhancement factor envisioned by McCray et al. [11]. It should be noted that parameters held within the inner parentheses equal the water/CD partitioning constant of McCray et al. [11].

Enhanced solubilization of organic-liquid mixtures

McCray and Brusseau [11] applied Raoult's Law to their earlier formula (36) to develop an equation to determine

the saturated levels of HOCs in a cyclodextrin aqueous solution when exposed to an excess liquid containing multiple HOCs. Equation shows their model where X_i is the mole fraction of the i th HOC component in a multi-component HOC liquid, S_{Ai} is the saturated concentration in the CD solution, and the other terms are specific to that HOC.

$$S_{Ai} = X_i S_{Wi} (1 + K_{C_{Wi}} C_{CD}) = X_i S_{Wi} E_i \quad (38)$$

While this equation does permit accurate estimation of the total amount of each component in solution, it again over estimates the HOC amount in the water portion of the liquid. Below is a version of Raoult's Law for CD solutions using the nomenclature of this paper.

$$S_{Ai} = X_i S_{Wi} \left(1 + \left(\frac{V_{mCD}}{K_{Ai23}} - V_{mCD} \right) C_{CD} \right) = X_i S_{Wi} E_i \quad (39)$$

HOC distribution in a water-CD-adsorbent porous media system

Gao and Blanford [32] found in batch studies that the presence of 2-hydroxypropyl-beta-cyclodextrin in aqueous solutions decreased the sorption trichloroethylen to granulated activated carbon (GAC). Specifically, they found that phase distribution between water and GAC followed a Freundlich isotherm.

$$C_{AS} = K_f \cdot C_{A2}^n \quad (40)$$

where C_{AS} is the adsorbed concentration of the HOC on the adsorbent ($\text{mol}_{\text{HOC}} \text{kg}_{\text{adsorbent}}^{-1}$), K_f is the Freundlich adsorption coefficient ($\text{L}_{\text{water}} \text{kg}_{\text{adsorbent}}^{-1}$), which indicates the adsorption ability and n is a constant indicative of adsorption affinity. From their batch studies, they found that if the HOC mass associated with CD was removed from consideration, then the parameters for the isotherm describing the distribution of the HOC between water and the adsorbent phases was statistically the same as the isotherm parameters in the absence of cyclodextrin. Thus, in the nomenclature of this paper, the adsorbent would function as another phase in the mass balance equation and the distribution between the water and adsorbent phase would follow Eq. 40 and thus Eq. 37 would become:

$$\frac{M_{AT}}{V_{liq}} = C_{A2} \left(1 + \left(\frac{V_{mCD}}{K_{A23}} - V_{mCD} \right) C_{CD} + K_f C_{A2}^{n-1} \frac{M_S}{V_{liq}} \right) \quad (41)$$

where M_S is the mass of the adsorbent. Over short ranges in concentration a linear isotherm is appropriate and Eqs. 40 and 41 further simplify.

HOC distribution in a system of water and multiple types of CD

Native and derivatized CDs have different affinities for various types of HOCs and metals and they have sharply different commercial costs. To take advantage of different affinities and reduce the expense of application, circumstances could exist where multiple CDs would be used to address a mixture of HOCs or a metal/HOC mixture. As an example Hoffman et al. [6] found that 2-carboxymethyl-β-cyclodextrin (CM-β-CD) mitigates the toxicity of cadmium, cobalt, and copper during biodegradation of naphthalene, but compared to HP-β-CD, CM-β-CD has a lower affinity for HOCs and is far more expensive [4, 5]. Thus, an application of both CDs may make sense. As a further example, 2-hydroxypropyl-γ-cyclodextrin has greater affinity for large polycyclic aromatic hydrocarbons (PAHs) than HP-β-CD, but is far more expensive [33]. Thus, a mixture of these two CDs may make sense for application to a coal-tar site where a range of PAHs are present.

If the CDs being chosen were known not to polymerize, then they would act independently and the equations for HOC distribution would be straight forward. The additional CDs would simply be additional phases and the distribution coefficients with water would have the same form as the first CD. Thus for the same system as Eq. 37, the addition of a multiple types CDs would follow the form of:

$$\frac{M_{AT}}{V_{liq}} = C_{A2} \left(1 + \sum_{i=3}^{n+2} \left(\frac{V_{mCDi}}{K_{A2i}} - V_{mCDi} \right) C_{CDi} \right) \quad (42)$$

where n is the number of distinct types of CDs. Since the process of derivatizing cyclodextrin results in a population of cyclic oligosaccharides with varying degrees of substitution, this equation also elegantly describes the complex behavior even of aqueous solutions containing a single type of modified CD.

Summation

The development of derivatives of native cyclodextrins in technical grades has resulted in a series of studies investigating the potential of CD as an agent to improve the remediation of hazardous waste sites. However, the variation in the purity and degree of substitution of derivatized CDs and a lack of reporting of this information has inhibited the development of models to uniformly assess the results of these studies. Because that information is becoming widely available, the above model provides a means to evaluate HOC/CD interactions on a molar basis

and improves the ability of environmental scientists to reproduce and compare their results.

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