

Math 122 Syllabus (SUMMER 2022 revision)
Submitted by: Elementary Courses Committee
Mathematics Department, Queens College

This course satisfies the Mathematical and Quantitative Reasoning (MQR) requirement of the Pathways General Education Required Core. Below is the Learning Outcomes that all MQR courses satisfy:

MQR 1: Interpret and draw appropriate inferences from quantitative representations, such as formulas, graphs, or tables.
MQR 2: Use algebraic, numerical, graphical, or statistical methods to draw accurate conclusions and solve mathematical problems.
MQR 3: Represent quantitative problems expressed in natural language in a suitable mathematical format.
MQR 4: Effectively communicate quantitative analysis or solutions to mathematical problems in written or oral form.
MQR 5: Evaluate solutions to problems for reasonableness using a variety of means, including informed estimation.
MQR 6: Apply mathematical methods to problems in other fields of study.

NOTES TO THE INSTRUCTOR

- Students have the options of either purchasing the loose-leaf format of the textbook, bundled with an Enhanced WebAssign* Printed Card Access, or an Enhanced WebAssign Instant Access code with an eBook. The electronic option is the cheaper of the two.
- Instructors are expected to use WebAssign for homework assignments. Students tend to complete the homework if it is counted for part (more than 5%) of their final grade. WebAssign homework should be designed by each instructor, to assess their students regularly. Careful selection, and a reasonable amount, of homework exercises are encouraged. Each assignment should align with the objectives for each lesson.
- The Teacher's Edition of the text provides a set of instructor resources (see Preface xi - xvi), which includes lesson plans. Instructors should use the *Alternate Example*, or examples from a supplemental source for class instruction. Students may want to refer to the textbook examples.
- Wherever possible, instructors should help students to become proficient with, and gain confidence from using, the graphing calculator. The Department approves the use of a TI 83 or 84. The authors of the textbook offer suggestions integrating the graphing calculator into the course.
- Three in-class exams, throughout the semester, are recommended. Time may be set aside for review sessions, inside or outside of the classroom. Instructors must pay keen attention to the pacing of the course (See suggestions for time allocation below).
- Instructors should ensure that students understand the core concepts, before class time is spent covering too many problems with high levels of difficulty. The idea is quality over quantity and difficulty.
- Students who take this course have interacted with some of this material before. They have many misconceptions, and preconceptions. Where possible, use what they know to motivate class discussions. Also, use what they did not learn well to discuss common errors and misconceptions. Give students opportunities to discuss and dispel these errors.
- Where appropriate, instructors should offer both algebraic and geometric meanings. For example, students should be able to solve the exponential equation $e^x = 2$ algebraically, and also use their calculators to find the point where the graphs of $y = e^x$ and $y = 2$ intersect.
- Instructors should provide opportunities for students to demonstrate that they have achieved the objectives for each lesson. Use the list of objectives below as a guide for lesson planning and creating assessments. Consider these objectives as a laundry list of things students should be able to do upon completion of Math 122. Instructors may provide students with a list of objectives as a guide to prepare for each exam.
- Students must have an opportunity to interact with word problems. This is a service course for calculus, the natural sciences and business courses. The authors offer many examples from different fields.
- Students must receive a course syllabus. See *Suggestions for an Effective Syllabus* on the QC Provost's Webpage.
- See attached suggested schedule.

* *Enhanced WebAssign* is an online learning program that includes problems from the text, tutorial videos, and eBook. Students receive immediate feedback on their work and have the option to try different versions of the same problem for additional practice. Tutorial resources such as *Master It* and *Watch It* provide step by step guidance to help students solve problems. *Chat About It* gives students 300 minutes of 24/7 tutorial assistance with their homework.

Math 122 Course Topics

THE FOLLOWING CHAPTER 1 SECTIONS OF THE SYLLABUS ARE TO BE TREATED AS REVIEW MATERIAL. INSTRUCTORS SHOULD COVER ONLY THE TOPICS LISTED ON THE SYLLABUS WITHIN EACH OF THE SECTIONS BELOW.

Instructors may create a handout with questions to guide the review of these topics. Students who struggle with these topics should be encouraged to get help outside of the classroom (Math Lab tutoring, online videos, etc.). A suggested review sheet is available for instructors to use.

Review the following topics and assess that students are able to:

1.1: Write sets in interval notation and classify real numbers. *There is a common misconception that real numbers are limited to integers. Often, students disregard zero and irrational-numbered solutions to equations as non-real.*

NOTE: It is advisable to show students the following definition of fraction addition, as it is extremely useful and arises in many examples & contexts within Pre-Calculus & Calculus: $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

1.2: Apply the laws of exponents to simplify expressions with exponents (zero, negative, and fractional exponents), write an expression with a fractional exponent in radical form. Provide opportunities for students to dispel misconceptions such as $\sqrt{3x^2} = 3x$, and $\sqrt[3]{2x^2} = (2x)^{2/3}$

1.3: Factor polynomials: Factor out common factors [include examples like *Example 6, (c)*], difference of two squares, factor quadratics, factor by grouping, factor completely.

1.4: Simplify rational expressions by factoring (Example 2); simplify compound fractions (Example 7)

NOTE: Discuss the following common errors. Assess whether students are able to identify these errors. The textbook contains the following table on page 42:

Correct Multiplication Property	Common Error with Addition
$(a \cdot b)^2 = a^2 \cdot b^2$	$(a + b)^2 \neq a^2 + b^2$
$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \ (a, b \geq 0)$	$\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$
$\sqrt{a^2 \cdot b^2} = a \cdot b \ (a, b \geq 0)$	$\sqrt{a^2 + b^2} \neq a + b$
$\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{a \cdot b}$	$\frac{1}{a} + \frac{1}{b} \neq \frac{1}{a + b}$
$\frac{ab}{a} = b \ (a \neq 0)$	$\frac{a + b}{a} \neq b$
$a^{-1} \cdot b^{-1} = (a \cdot b)^{-1}$	$a^{-1} + b^{-1} \neq (a + b)^{-1}$

1.5: Solve quadratics: Use quadratic formula. Include examples for which $b = 0$ or $c = 0$, for a quadratic equation in the form $ax^2 + bx + c = 0$. *If students are comfortable with factoring, then they may use this method, too; however, instructors should not spend time teaching students how to factor polynomials at this point. Students should be reminded of the geometric meaning of solving quadratic/polynomial equations. Defer review of Completing the Square until needed in Chapter 3.*

1.8: Solve quadratic inequalities. *Students may not have seen this topic before, so instructors may spend some time discussing a few examples. Students may need this for Calculus 141.*

1.9: Write the equation of a circle centered at (h, k) with radius r . *Helpful for unit circle trig.*

1.10: Define slope, discuss characteristics of a line based on slope (e.g. positive slope means that line is increasing from left to right, negative slope, zero slope, no slope), determine the slope of a line given two points, write the equation of a line, write the equation of vertical & horizontal lines. *Omit Parallel and Perpendicular lines.*

COURSE OBJECTIVES

CHAPTER TWO

Section 2.1

- Define function
- Discuss the different representations of a function (graph, table, words, etc.)
- Define domain and range of a function
- Discuss how to evaluate a function and use the graphing calculator to evaluate functions as well. *Students should define y as a function of x , so that $f(x)$ is the value of y at x .*
- Simplify the difference quotient for a function (*Example 5*) – *tell students that the difference quotient represents the average rate of change of f , to be discussed in calculus*
- Discuss piecewise-defined functions (These come up in Calculus I often in context of Limits & Continuity)
- Demonstrate how to evaluate a piecewise-defined function
- Sketch the graph of a piecewise-defined function. *It is an option to discuss the idea of continuity when applicable.*
- Determine the domain of a function (*Example 7*)

Section 2.2

- Establish the definition of a graph of a function. Make sure students understand the distinction between a function and its graph
- Use the graphing calculator to graph functions (*use the ZOOM feature, adjust WINDOW*)
- Discuss, in detail, the graphs of $f(x) = x^n$ for positive n , odd and even, and explain how the graphs are similar or different as n increases. Also, discuss, in detail, the graphs of all other “parent functions” shown on page 166.
- Apply the vertical line test to determine whether a graph represents a function

Section 2.3

- Determine the domain and range of a given graph
- Use graphing calculator to solve equations graphically
- Define maximum and minimum values of a function
- Use the graphing calculator to determine the local min/max of a function

Section 2.6

Review the graphs of all “parent functions” covered in section 2.2 (see page 166)

- Sketch graphs using transformation of basic functions
Note: Shrinking and stretching of graphs are difficult to illustrate for some functions. Describe what happens to the graphs of function in these cases, but students will need to find intercepts to help show that a shrink or stretch occurred. Review/discussion of solving certain equations (absolute value, rational, etc.) may be necessary.

Section 2.7

- Combine and simplify functions using order of operations. *This should be a quick review. Instructors should use this opportunity to address common mistakes and misconceptions, for example, $(x + y)^2 = x^2 + y^2$.*
- Use composition notation, $(f \circ g)(x) = f(g(x))$.
- Find the composition of functions
- Recognize a composition of functions (function decomposition)
Note: This topic is extremely important for students planning to take Calculus since computing derivatives using the Chain Rule requires students to recognize composition of functions

Section 2.8

- Define one-to-one functions
- Use horizontal line test to determine whether a function is one-to-one
- Determine whether a function (graph or equation) is one-to-one
- Define the inverse of a one-to-one function
- Determine the inverse of a one-to-one function
- Discuss modifying the domain of a function, such as $y = x^2$, to obtain a one-to-one function
- Discuss the notation for inverse function and *discuss the misconception that $f^{-1}(x) = \frac{1}{f(x)}$*
- Sketch the graph of a one-to-one function $f(x)$, then graph its inverse function by reflecting the graph of $f(x)$ about the line $y = x$.

Modeling with functions (page 237) - Note: This is an extension of the Chapter 2 topics of the syllabus

- Write and analyze a mathematical model. *Students need not find minimum/maximum values here. The goal of this section is to prepare students for the topics of optimization and related rates in calculus. Include questions to provide opportunities for students to review formulas for area/perimeter/volume/surface area of 2D and 3D objectives, accordingly.*

CHAPTER THREE

Section 3.1

- Establish the definition of quadratic function
- Express a quadratic function in standard form [*Instructors should assess, by reviewing, that students know how to complete the square. Students may also find it helpful to check on the graphing calculator that the graphs of the general and standard forms of a quadratic function are the same.*
Optional: By completing the square on $f(x) = ax^2 + bx + c$, in conjunction with their knowledge of transformations, students will get a chance to see why $x = \frac{-b}{2a}$ is the x-value of the vertex.
- Determine whether the vertex is a minimum or maximum point based on the sign of the leading coefficient
- Determine the minimum or maximum value of a quadratic function, given the function or its graph. *Include applications as a follow up to the section on modeling; cover business applications where students have an opportunity to, say, maximize profit or minimize revenue – this will be helpful in Math 131.*
- Determine the coordinates of the vertex
- Write an equation for the axis of symmetry
- Sketch the graph of a parabola using transformations of the parent function $y = x^2$ and the standard form
- Determine the domain and range of a quadratic function
- Determine the x-and y-intercepts of a quadratic function
- Use the graphing calculator to graph quadratic functions, determine their roots, and minimum/maximum values.

Section 3.2

- Establish the definition of a polynomial function. *Instructors may need to review the definition of integer, so that students understand the term ‘integer exponents’. Discuss a polynomial as smooth and continuous.*
- Identify the degree, leading term, and leading coefficient of a polynomial
- Use transformation, to sketch the graph of basic polynomial functions. *Review the graphs of the parent functions*
- Determine the end behavior of a polynomial by comparing to that of power functions with even or odd degree
- Determine the domain and range of a polynomial
- Use the graphing calculator to determine the value of a polynomial at a given x value
- Use the graphing calculator to determine the zeros of a polynomial function. *Instructors should show students how to adjust the viewing window on the screen, then help students to use the table of values to identify the appropriate viewing window for the graph of a given function.*
- Determine the maximum and minimum values of a polynomial function (when they exist) given the graph. Use the calculator to determine these values where appropriate. Discuss the difference between maximum/minimum values and local extrema.
- Show that if $x = a$ is a zero of $P(x)$, $P(a) = 0$ and $(x - a)$ is a factor of $P(x)$ [Criteria on page 259]. *Discuss multiplicity of zeros and the shape of the graph near the zeros*
- *Sketch the graph of a polynomial using the concepts discussed in this section and using the graphing calculator as a tool to guide the sketch. (Label the coordinates of the extrema & intercepts)*

Section 3.3

- Briefly review the topic of Long Division of Polynomials (Recall this is a Math 115 topic)
- Introduce Synthetic Division of Polynomials
- *Apply either long or synthetic division and sketch the graph of a polynomial using the concepts discussed in section 3.2*
- Omit the Remainder Theorem
Note: Students should be able to perform either long or synthetic division of polynomials, extending their knowledge of how to factor polynomials and sketch graphs based on the zeros and intercepts of the polynomial [Note: Cases where the remainder is non-zero should be addressed].

Section 3.6

- State the definition of a rational function as a ratio of polynomials so that certain concepts related to polynomials can be applied
- Determine the domain of a rational function using its function rule
- Determine the domain and range of a rational function from its graph
- Determine the intercepts of a rational function
- Discuss the graph of basic rational functions using transformations of $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$
- Establish the definition of vertical and horizontal asymptotes
- Determine the equations of the vertical and horizontal asymptotes. *Instructors may guide students on how to use the graphing calculator and its tables of values to determine asymptotes of a rational function.*
- Determine the end behavior of rational functions near the asymptotes by analyzing graphs
- Use the graphing calculator to sketch rational functions. *Instructors must discuss pitfalls of the graphing calculator here, for example, asymptotes not visible.*
- Option: Discuss Slant Asymptotes

CHAPTER FOUR

Section 4.1

- State the definition of an exponential function with base b . Discuss the differences between the base b being greater than 1 and the base b being in between 0 and 1.
- Discuss the properties of the basic exponential function $f(x) = b^x$. *Instructors should include discussion about the domain and range, horizontal asymptote, and discuss $f(x) = b^x$ as a parent function for each possible choice of the base b , so that students understand how to apply transformations to the graph of any exponential function*
- Evaluate exponential functions
- Sketch the graph of basic exponential functions using transformations
- Sketch the graph of basic exponential function using the graphing calculator (*use calculator to determine the zeros of the exponential functions where appropriate*)
- Solve compound interest problems

Section 4.2

- Define the natural exponential function. *Instructors may have a brief discussion about the number e .*
- Evaluate the natural exponential function
- Sketch the graph of the natural exponential function and other variations of this function using transformations
- Solve compound interest problems where interest is compounded continuously

Section 4.3

- State the definition/notation of a logarithmic function. *It is important for students to interpret the logarithm with base b as the inverse of an exponential function with base b*
- Convert between the logarithmic form and exponential form of an equation
- Evaluate logarithms (include logarithms of various different bases as well as base 10 and base e)
- State and apply properties of logarithms
- Sketch the graphs of logarithmic functions using transformations. *Instructors should include discussion about the domain and range, thus vertical asymptote. For basic graphs, students may determine the intercepts by using the definition of logarithm to solve equations.*
- Interpret the natural logarithmic function as the inverse of the natural exponential function, which is one-to-one
- State properties of natural logarithms

Section 4.4

- Establish the laws of logarithms.
- Apply the laws of logarithms to expand and combine logarithmic expressions.
- Use the laws of logarithms to evaluate logarithmic expressions.
- Use the change of base formula to evaluate logarithms. *Newer graphing calculators allow for calculating logs in bases other than 10 and e .*
Note: The change of base formula for logarithms is very useful in computing the value of certain logarithmic integrals in Calculus II
- Use the calculator to evaluate logarithms.
Note: *Instructors may want to reinforce the usefulness of the change of base formula for logarithms here, which is helpful to students with older models of the graphing calculator.*

Section 4.5

- Solve exponential equations. *Instructors should show students how to solve basic exponential equations that do not necessarily require calculator use, but also include equations that can be solved using the calculator.*
- Solve logarithmic equations
- Solve problems involving exponential equations

Unit circle trigonometry is useful for calculus. The following outline for trigonometry is a suggested guide for how instructors may cover chapters 5 and 6 simultaneously; however, instructors have the option of covering either Chapter 5 or 6 (in the interest of time).

Sections 5.1 and 6.1

- State the definition of an angle in standard position
- Discuss how to interpret angles using radian measure
- State the definition of coterminal angles and find coterminal angles of a given angle in standard position
- Determine the length of a circular arc; Recall the equation of the unit circle
- Determine the coordinates for terminal points on the unit circle (i.e., given one coordinate, determine the other, using the equation of the unit circle).
- Determine the coordinates of the terminal points associated with radian measure in each quadrant. *It is helpful to introduce definition of degree measure and radian measure and show how they are related, here. Usually, students know that the total degree measure in a circle is 360° . Define the radian measure as the distance traveled when the degree measure is θ° , and relate distance traveled around the circle to circumference. Find the circumference of the unit circle when the degree measure is 360° . As such, 360° corresponds to 2π radians, whose terminal point is $(1,0)$. It follows that 180° is π radians, and the terminal point is $(-1,0)$...and so on. The textbook shows how to evaluate the terminal points for 30, 45, 60 degrees. Do this for the angles in the first quadrant, and reflect the terminal points about these angles in the other 3 quadrants. For example, 210° is the reflection of 30 degrees in the 3rd quadrant. See attached].*
- Convert between degrees and radians. *It is helpful for students to understand that the radian measure corresponds to the arc length when the degree measure is θ°*
- Define reference angle and determine the reference angle for a given angle
Omit Linear and Angular Speed

Sections 5.2 and 6.2

- Establish the definitions of the trigonometric functions *as functions where the function input is an angle, and the function output is a real number.*
- Evaluate trigonometric functions of special angles using the unit circle. *When students interpret the sine of the angle of the y-value of the terminal point that corresponds to the angle, then they are able to see that, for example, $\sin 30^\circ = \frac{1}{2}$ or $\sin 210^\circ = -\frac{1}{2}$.*
- Establish the numerical signs of trig functions in each quadrant. *Allow students to determine the sign of the trig functions in each quadrant by relating this to the coordinates of the terminal points.*
- Establish the fundamental trigonometric identities. *Instructors may introduce the Pythagorean identities, by using the equation of the unit circle, and the definition of the trig functions. This is an extremely important topic for students moving on to Calculus.*
- Define the trig ratios as ratios of the sides of a right triangle, with acute angle θ .
- Given one trig ratio, use the right triangle and Pythagorean theorem to find the other five ratios
Show students how the unit square, and equilateral triangle are used to establish the Special Right Triangles and the trig ratios for reference angles measuring 30, 45 and 60 degrees.

Section 6.3

- Evaluate trig functions for any angle using reference angles and the signs of trig functions in each quadrant
- Use the calculator to evaluate trig functions. *Attention must be given to the MODE.*

Section 5.3

- State properties of the basic trigonometric functions (domain, range, amplitude, period)
- Sketch the graph of the basic trigonometric functions; Revisit the concept of Parent Functions here
- Sketch the graph of basic trig functions using transformations. Include amplitude and phase shift, as well as vertical shifts and reflections; Relate this to transforming the graphs of parent functions.
- Sketch the graph of trig functions in the form $y = A \sin(Bx + C) + D$, $y = A \cos(Bx + C) + D$.

Section 6.4

- Establish the properties and notation of the inverse trig functions (*domain, range*). *It is helpful for instructors to make reference to one-to-one functions here, and thus the need for restrictions on the domains of inverse functions; identify the appropriate quadrants for the outputs.*
- Evaluate inverse trig functions
- Evaluate composition of trig functions and their inverses

NOTE: Instructors should not spend too much time covering sections 6.5 & 6.6, but they should be discussed.

Section 6.5

- Discuss triangle congruence using ASA, SAA, SSA, SAS, and SSS
- Introduce the Law of Sines as related to triangle congruence
- Show the possible outcomes of applying Law of Sines to various triangles. *Omit the ambiguous case*
- Show an example of an application of Law of Sines

Section 6.6

- Recall triangle congruence using ASA, SAA, SSA, SAS, and SSS
- Introduce the Law of Cosines as related to triangle congruence
Show examples where students must solve for either a missing side or a missing angle of a triangle using Law of Cosines. Show an example of an application of Law of Cosines.
Optional topic: Heron's Formula for the area of a triangle using semiperimeter.

NOTE: Chapter 7 material should be presented in detail with true learning goals in mind, as this section is important in students' preparation for Calculus. Students who plan to enroll in any Calculus sequence will encounter much of the material discussed in Chapter 7, in particular. It is important that students not only understand trigonometric functions and their properties, but they must know how the trigonometric functions are related to each other via identities.

Section 7.1

- Use known trigonometric identities to simplify trigonometric expressions, emphasis on converting to sine and cosine when appropriate
- Deduce well-known and useful identity equations.
- Prove trigonometric identities.

Section 7.2

- Establish the addition and subtractions formulas for trig functions (*Tangent formulas are optional*)
- Use the addition and subtraction formulas to determine the value of trigonometric functions by rewriting angle measures as sums or differences of the angles associated with the special right triangles
Instructors should dispel the misconception that $\sin(A + B) = \sin A + \sin B$.
- Disregard the topic of Sums of Sines and Cosines (Meaning expressions of the form $A \sin x + B \cos x$)

Section 7.3

- State the double-angle formulas for sine and cosine (Formula for $\tan(2x)$ is optional)
Suggestion: Demonstrate for students how the double-angle formulas are derived by using the sum and difference formulas for sine and cosine.
- Demonstrate the use of the double-angle formulas to evaluate trigonometric expressions given certain conditions (See example 1 on page 554 of the course textbook). Prove identity equations that involve double-angle formulas
- State the power-reducing formulas for sine and cosine (tangent formula is optional)
- Demonstrate the use of the power-reducing formulas to rewrite trigonometric expressions in different forms
Suggestion: *Demonstrate for students how to derive the power-reducing formulas using formulas for $\cos(2x)$. Power-reducing formulas are used extensively in Calculus II and III and are often incorrectly referred to as half-angle formulas.*
- Disregard all half-angle formulas and disregard product-to-sum and sum-to-product formulas

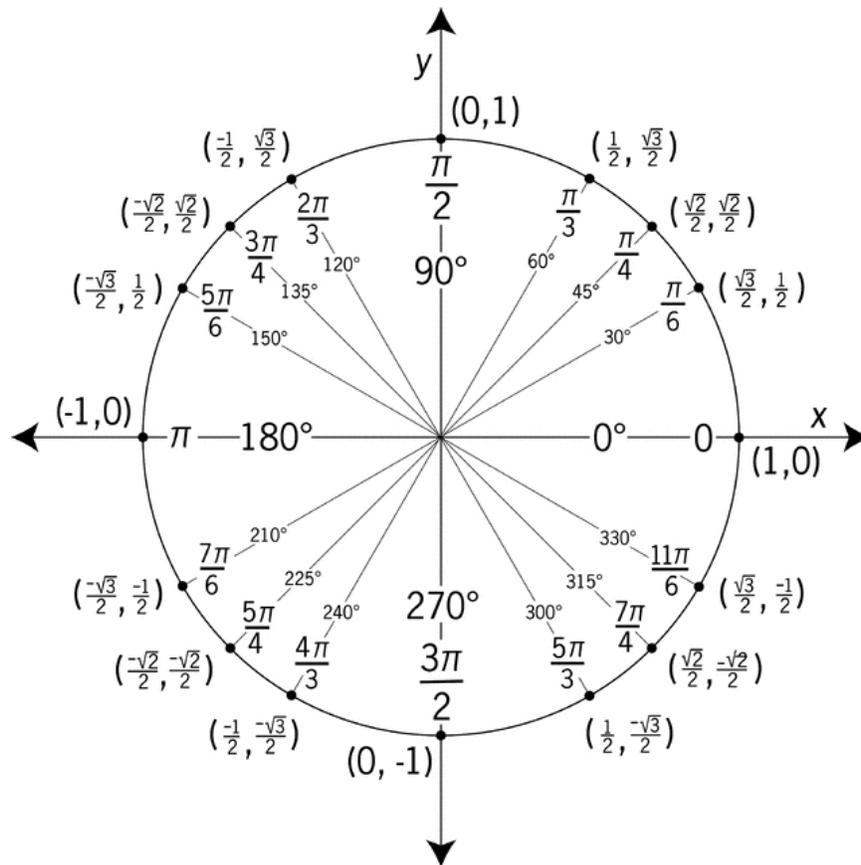
Section 7.4

- Solve basic trig equations. *Suggestion: Illustrate the steps taken when solving a basic linear equation in the variable x , then compare this process to that of solving a basic trig equation, which incorporates knowledge of trig functions at commonly used angle measures.*
- Solve equations of the linear and quadratic type, as well as those that need to be solved by factoring. *Show examples with general solutions as well as solutions on a closed interval such as 0 to 2π . Dispel misconceptions such as $\sin^2 x = (\sin x)(\sin x) \neq \sin(x^2)$ so that students are able to factor and solve when trig equations are "quadratic" in nature. Discuss cases where an answer may be rejected when a value lies outside the range of a trig function. Make the connection between inverse trigonometric functions and problem solving.*

Section 7.5

- Solve trig equations that require the use of identities. *Include the Pythagorean Identities as well as double-angle formulas. Show examples with general solutions as well as solutions on a closed interval such as 0 to 2π .*

Unit Circle Trigonometry



Instructors may give students a copy of the unit circle, without the terminal points and angle measures. As a class activity, once students learn how to convert from degrees to radians, and the terminal points have been discussed, students may complete the unit circle as seen above. The textbook shows how the Pythagorean Theorem may be used to find the coordinates of the terminal points.

Suggested Schedule

There are 28 class sessions per semester, each 1 hour 50 minutes. By giving 3 exams, this leaves 25 class sessions for instruction and review. The following is a guide for how instructors may pace the course.

- **Chapter 1 review materials & all sections for Chapter 2 on this syllabus, including the section on Modeling with Functions (page 237 of course textbook)**
Suggestion: Allocate 8 class sessions, which should include time to review for an exam
- **All sections for Chapters 3 and 4 on this syllabus**
Suggestion: Allocate 8 class sessions, which should include time to review for an exam
- **All sections for chapters 5, 6, and 7 on this syllabus**
Suggestion: Allocate 8 class sessions, which should include time to review for an exam

This leaves one remaining class session for additional instruction. Suggestion: Allocate this additional time to Trigonometry or perhaps use for final exam review.

MATH 122 Pre-Calculus
Chapter 1 Topics Review Assignment

Categorizing Real Numbers (1.1)

1. Given the set $\left\{-3, \frac{4}{3}, \sqrt[3]{8}, 0.12, 0, \sqrt{2}, 2.151515 \dots, \pi, -17, \frac{1}{\pi}, \sqrt{-5}\right\}$, determine which numbers in the set are:
- Natural Numbers
 - Rational Numbers
 - Irrational Numbers
 - Real Numbers
 - Non-real numbers

Interval Notation (1.1)

2. Write the following inequalities using interval notation:
- a) $-1 \leq x < 5$
 - b) $1.5 \leq x \leq 4$
 - c) $-8 < x$
 - d) $x > \frac{4}{3}$
 - e) $x \leq 10$
3. Write the following intervals as an inequality:
- a) $[0, 18]$
 - b) $(-\infty, -6)$
 - c) $\left[\frac{5}{2}, \pi\right)$
 - d) $\left[\frac{5}{2}, \infty\right)$
 - e) $(0.1, 21]$
 - f) $(-4, +\infty)$

Simplifying Expressions with Exponents and Radical Expressions (1.2)

4. Simplify the expressions below. Write final answers using positive exponents where appropriate.
- a) $x^{-3}x^7$
 - b) $(x^{-3})^2$
 - c) $(2x)^4$
 - d) $(2x)^{-4}$
 - e) $\frac{c^9}{c^5}$
 - f) $\frac{6^0}{6^2}$
 - g) $\left(\frac{x}{2}\right)^5$
 - h) True/False: $\left(\frac{4}{5}\right)^3 = \left(\frac{5}{4}\right)^{-3}$
5. Simplify the expressions below.
- a) $(2a^3b^2)(3ab^4)^3$
 - b) $\left(\frac{x}{y}\right)^3\left(\frac{y^2x}{z}\right)^4$
 - c) $(8x^2)^2(-5x^0y^2)^3$
 - d) $(-37ax^{41})^0$

6. Simplify the expressions below. Write the final answers using positive exponents where appropriate.
- $\frac{6xy^{-4}}{2x^{-2}y^2}$
 - $\left(\frac{y}{3x^3}\right)^{-2}$
 - $\left(\frac{x^{-3}}{y^{-1}}\right)^{-2}$
7. Rewrite each of the following terms as radical expressions.
- $4^{\frac{1}{2}}$
 - $8^{\frac{2}{3}}$
 - $125^{-1/3}$
 - $x^{1/4}$
 - $(2x)^{1/2}$
 - $(-27)^{1/3}$. Is this a real number?
8. Write each radical expression using rational exponents (fractions).
- \sqrt{y}
 - $\sqrt[3]{x^5}$
 - $\sqrt{3x+1}$
 - $\sqrt[6]{x^4}$
 - $\sqrt[6]{(5x)^3}$
9. True/False: Determine if each equation below is true or false, and justify your answer.
- $\sqrt{3x^2} = 3x$
 - $\sqrt[3]{2x^2} = (2x)^{2/3}$
 - $\sqrt[n]{x^n} = 1$, whenever $n \geq 2$

Factoring Polynomials and Simplifying Rational Expressions (1.3 & 1.4)

10. Factor the following expressions using the indicated method above each set of examples.

Greatest Common Factor (GCF)

- $3x^2 - 6x$
- $-2x^4 + x$
- $14x - 35x^5$
- $8x^4y^2 + 6x^3y^3 - 2xy^4$
- $(2x + 4)(x - 3) - 5(x - 3)$

Trinomial Factoring

- $x^2 - 6x + 9$
- $x^2 + 6x + 9$
- $x^2 - 7x + 10$
- $y^2 - 2y - 63$
- $x^2 - 14x + 48$
- $2x^2 + 5x + 3$
- $6x^2 + 7x - 5$

Difference of Perfect Squares

- a) $x^2 - 16$
- b) $4x^2 - 25$
- c) $a^4 - b^6$
- d) $36 - y^2$

Factor Completely

- a) $3x^3 - 27x$
- b) $x^4 - 1$
- c) $2x^2 + 10x + 12$

Factor by Grouping

- a) $x^3 + x^2 + 4x + 4$
- b) $x^3 - 2x^2 - 9x + 18$

Simplify each Rational Expression

a) $\frac{x^2 - 1}{x^2 + x - 2}$

b) $\frac{x^2 + 4x + 4}{x^2 + 3x + 2}$

c) $\frac{x^3 - 8}{x^3 - 2x^2}$

d) $\frac{8 - 2x}{x^2 - x - 12}$

e) $\frac{2x^3 - x^2 - 6x}{2x^2 - 7x + 6}$

f) $\frac{\frac{1}{2} + \frac{3}{x}}{\frac{x+3}{4}}$

g) $\frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

Solving Quadratic Equations (1.5)

11. Solve the following quadratic equations

- a) $x^2 - 5x + 6 = 0$
- b) $x^2 + 14x + 48 = 0$
- c) $x^2 + 5x = 24$
- d) $x^2 - 5 = 0$
- e) $(x - 4)^2 = 5$
- f) $(x + 2)^2 = 16$
- g) $2x^2 - x - 3 = 0$

Solving Quadratic Inequalities (1.8)

12. Solve the following quadratic inequalities. You may use a number line as a guide, but your solutions should be written using interval notation.

- a) $x^2 \geq 9$
- b) $x^2 < 4$
- c) $x^2 - 3x - 18 \leq 0$
- d) $x^2 \leq 5x - 6$
- e) $x^2 > -(x - 6)$
- f) $2x^2 + 7x \geq 0$

Equation of a Circle (1.9)

13. Follow the directions for each set of examples.

Write the equation of the circle with the indicated center and radius.

- a) Center $(3, -1)$, $r = 3$
- b) Center $(0, 0)$, $r = \sqrt{5}$
- c) Center $(-4, 1)$, $r = 1$

Determine the center and radius of the circle given by each equation below. Express the radius in simplest radical form where appropriate.

- a) $(x - 1)^2 + y^2 = 36$
- b) $(x + 2)^2 + (y + 7)^2 = 12$
- c) $(x - 3)^2 + (y + 3)^2 = 3$

Lines (1.10)

14. Follow the directions for each set of questions.

Find the slope of the line that passes through the given points.

- a) $(2, 3)$ and $(7, 4)$
- b) $(-1, 5)$ and $(6, -2)$

Write the equation of the line that satisfies the given conditions.

- a) The line has a slope of 3 and a y-intercept of -2 [Note: Sketch the graph of this line].
- b) The line has a slope of $\frac{2}{5}$ and y-intercept of 4
- c) The line passes through the points $(-1, 2)$ and $(3, -4)$ [Note: Write the equation in both Point-Slope form and in Slope-Intercept form]

Sketch the graphs of line given by the equations below.

- a) $x = -3$
- b) $y = 1$
- c) $x = 4$
- d) $y = -2$