PHYS 8 - The Science of Fractals and Its Applications

General Information:

College: Queens College

Department: Physics

Course section, Day and Time of Class Meetings: TBD (2 meetings/week)

Building and Room Number: Remsen 209

Instructor name and contact information:

Larry S. Liebovitch, PhD

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Course Description:

Content: Fractals are physical or mathematical objects with an ever larger number of ever smaller pieces. This course shows how scientists use fractals to analyze and solve problems. It shows how mathematics can give new insights into understanding physical, biological, and social systems.

Pedagogy: Students learn in different ways. This course uses the tools of spreadsheets, graphs, algebra, numerical methods, folding pieces of paper, and performing in-class experiments to learn mathematical concepts and apply them to give insights into the nature and properties of physical, biological, and social systems.

History: The course was developed and taught at Florida Atlantic University by Dr. Liebovitch with the support of two grants from the National Science Foundation and those curricula materials are currently available on-line. It was very successful in attracting students who were previously fearful or unexperienced in mathematics to learn about science and mathematics. It also attracted some of the best undergraduate students who wanted a more innovative course in science and mathematics. The success of the course was evidenced by its increasing enrollment every year and outstanding student evaluations.

Textbook Information:

Required: The Mathematics and Science of Fractals by L. S. Liebovitch and L. A. Shehadeh, 2003. Free digital web materials, available on-line. These materials were created with the support of National Science Foundation grants: NSF DUE-9752226 and NSF DUE-9980715.

Recommended: Fractals and Chaos: Simplified for the Life Sciences, L. S. Liebovitch, Oxford University Press, 1998.

Attendance Policy:

Class participation is encouraged as many of the classes will be activities done in groups during class time. Video recordings of lectures and class activities will also be posted asynchronous on-line.

Discipline/Course Specific Learning Objectives:

List all discipline or course specific objectives that students can expect to learn in the course (these are *not* the general education learning objectives). It is recommended that learning objectives be numerically listed (e.g., LO1., LO2)

LO 1: **MQR 4, MQR 6**: Accomplishment. and Confidence: Science and mathematics is something that I can do.

LO 2: **MQR 1, MQR 2, MQR 3, MQR 5**: Fractals. Many things in the natural world consists of a few big pieces, many medium sized pieces, and a huge number of tiny pieces.

LO 3: **MQR 2, MQR3**: Isomorphisms. Given a problem that is hard to solve, we can sometimes find an equivalent problem that is easier to solve by different methods and we can solve the easier problem.

LO 4: **MQR 1, MQR 2, MQR 3, MQR 5**; Generalization. We can start with a concept and keep expanding it. For example, we can start with dimensions 1, 2, and 3 and then expand the concept of dimension to include dimensions like 0 and 6, and then expand it further to include dimensions like 1.261859507… or 1.584962501… .

LO 5: **MQR1, MQR 2, MQR 4, MQR 5, MQR 6**; Mathematics is the language of science. Mathematical concepts can help us better understand and make sense of physical, biological, and psychological worlds around us. The natural world can also provide insights into developing new mathematical concepts.

CUNY COMMON CORE
(do not modify the below statement – this statement must be included on all QC MQR courses)

All MQR courses must meet the following learning outcomes:

MQR 1: Interpret and draw appropriate inferences from quantitative representations, such as formulas, graphs, or tables.
MQR 2: Use algebraic, numerical, graphical, or statistical methods to draw accurate conclusions and solve mathematical problems.
MQR 3: Represent quantitative problems expressed in natural language in a suitable mathematical format.
MQR 4: Effectively communicate quantitative analysis or solutions to mathematical problems in written or oral form.
MQR 5: Evaluate solutions to problems for reasonableness using a variety of means, including informed estimation.
MQR 6: Apply mathematical methods to problems in other fields of study.

Examples of Classroom Activities

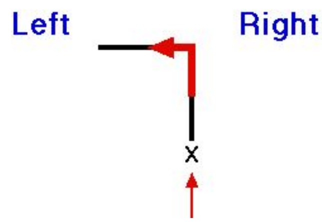
Lecture numbers refer to the Lectures in The Mathematics and Science of Fractals by L. S. Liebovitch and L. A. Shehadeh, 2003.

Activity Type #1, MQR 2, MQR 3, LO 3, LO 4. Translate a physical process described in natural language into a symbolic mathematical system and use iterative rules to predict the results of that natural process. Here the process is the iterative folding of a strip of paper and its representation is that of a symbol sequence.
Lecture 6: Using Symbols to Represent Paper Folds: L's, R's
Lecture 7: Paper Folding Iterations: Real Folds in Real Paper and the Symbols L and R

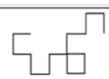
Objectives:

- We see that we can use symbols to represent things.
- We can represent the folds in a real piece of real paper by the sequence of symbols of "L" and "R".
- These symbols can make it a lot easier to figure out the properties of the real things.
- We see that repeated operations can make patterns whose little pieces are smaller copies of the whole thing

A strip of paper is folded n times, then unfolded. This is repeated for n = 1, 2, 3, etc.
Each fold is identified as LEFT or RIGHT



Many folds form a pattern that can also be represented by the letters L and R

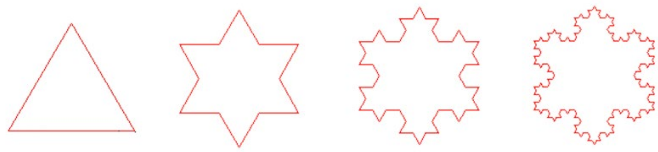
Iteration	Shape	Symbols representing the folding pattern
0	—	
1		L
2		LLR
3		LLRLLRR
4		LLRLLRRLLLRRLRR

Sometimes its easier to figure out the pattern from the actual paper folds, sometimes it's easier to figure out the pattern from the symbols. For many folds it's much easier to figure out the paper from the rules of the iterations of the symbols. Students are tasked with determining two different rules of how the symbol sequence changes with each iteration, with help from the instructor as needed.

Activity Type #2, MQR 2, MQR 5, LO 2, LO 4. Using the Koch curve to understand limits and mathematically solve for its area and perimeter length using analytical and numerical methods.
Lecture 10: The Koch Curve: Introduction to the Snowflake
Lecture 11: The Koch Curve: Area and Limits
Lecture 12: The Koch Curve: Zeno's Animals and the Area of the Koch Curve
Lecture 13: The Koch Curve: Length, Infinity, and Dimension
Objectives:

- What is the Area inside this snowflake?
- What is the Length around the perimeter of this snowflake?
- We see how to do a careful accounting of how much area is added at each iteration in making the Koch curve.
- We see that even though we keep adding new area at each iteration, the amounts added are so small that the total area reaches a limit.
- We see how to do a careful accounting of how the total length of the Koch Curve changes with each iteration.
- We see that the length of the Koch Curve gets larger and larger, forever! We see that we can use symbols to represent things.

The Koch curve is generated by replacing each line segment of length L by 4 pieces each of length L/3. Students are tasked with determining how the area and length depend on the number of iterations using analytical methods (algebra) and numerical methods (spreadsheet).



Area:

Step	Area of each new, smaller triangle added at this step	Number of new, smaller triangles added at this step	Area added at this step
1	0.4330	1	0.4330
2	(1/9)(0.4330)	3	0.1443
3	(1/9) (1/9) (0.4330)	4 x 3	0.0642
4	(1/9) (1/9) (1/9) (0.4330)	4 x (4 x 3)	0.0285

Step	Area of each triangle	Number of triangles	Area added at this step	Total area
1	0.4330127018922190000	1	0.43301270	0.433012702
2	0.0481125224324688000	3	0.14433757	0.577350269
3	0.0053458358258298700	12	0.06415003	0.641500299
4	0.0005939817584255410	48	0.02851112	0.670011424
5	0.0000659979731583934	192	0.01267161	0.682683034
6	0.0000073331081287104	768	0.00563183	0.688314861
7	0.0000008147897920789	3072	0.00250303	0.690817896
8	0.0000000905321991199	12288	0.00111246	0.691930355
9	0.0000000100591332355	49152	0.00049443	0.692424782
10	0.0000000011176814706	196608	0.00021975	0.692644527
11	0.0000000001241868301	786432	0.00009766	0.692742191
12	0.0000000000137985367	3145728	0.00004341	0.692785598
13	0.0000000000015331707	12582912	0.00001929	0.69280489
14	0.0000000000001703523	50331648	0.00000857	0.692813464
15	0.0000000000000189280	201326592	0.00000381	0.692817274
16	0.0000000000000021031	805306368	0.00000169	0.692818968
17	0.0000000000000002337	3221225472	0.00000075	0.692819721
18	0.0000000000000000260	12884901888	0.00000033	0.692820055
19	0.0000000000000000029	51539607552	0.00000015	0.692820204
20	0.0000000000000000003	2.06158E+11	0.00000007	0.69282027

Length of perimeter

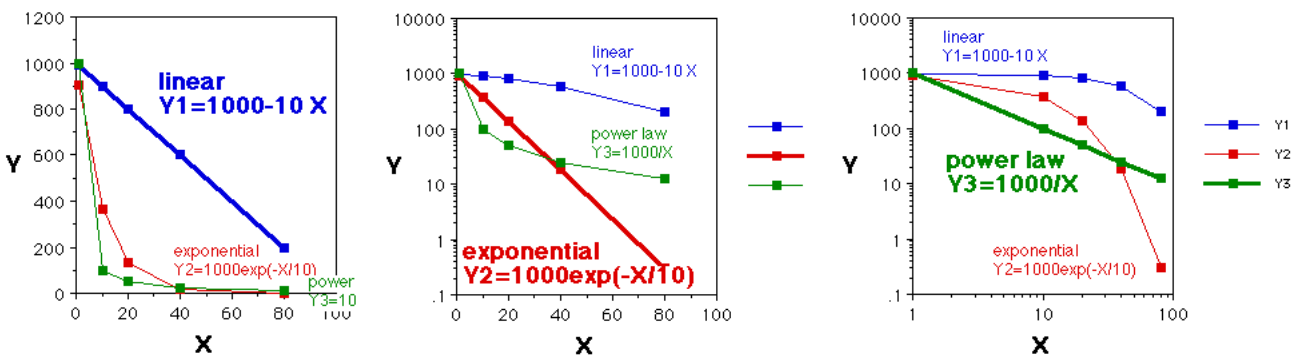
Step	Length of each line at this step	Number of lines at this step	Total length at this step
1	1	3	3.00
2	(1/3) (1)	4 x 3	4.00
3	(1/3) (1/3) (1)	4 x (4 x 3)	5.33
4	(1/3) (1/3) (1/3) (1)	4 x (4 x 4 x 3)	7.11

Step	Length of each line	Number of lines	Length added	Total length
1	1.0000000000	3	3.00	3.00
2	0.3333333333	12	1.00	4.00
3	0.1111111111	48	1.33	5.33
4	0.0370370370	192	1.78	7.11
5	0.0123456790	768	2.37	9.48
6	0.0041152263	3072	3.16	12.64
7	0.0013717421	12288	4.21	16.86
8	0.0004572474	49152	5.62	22.47
9	0.0001524158	196608	7.49	29.97
10	0.0000508053	786432	9.99	39.95
11	0.0000169351	3145728	13.32	53.27
12	0.0000056450	12582912	17.76	71.03
13	0.0000018817	50331648	23.68	94.71
14	0.0000006272	201326592	31.57	126.28
15	0.0000002091	805306368	42.09	168.37
16	0.0000000697	3221225472	56.12	224.49

Activity Type #3, MQR 1, MQR 2. Drawing inferences from quantitative representation of data as graphs.
Lecture 18: What Graphs Tell Us: Discovering Straight Lines
Lecture 19: What Graphs Tell Us: What the Straight Lines Mean
Objectives:

- We see how to use graphs to determine the mathematical relationships in data.
- We see how to figure out linear, exponential, and power law relationships.
- We see that we can measure change by the absolute change or the relative change.
- We see that the shapes of the graphs reveal their meanings.
- We see the meaning of linear, exponential, and power law relationships.

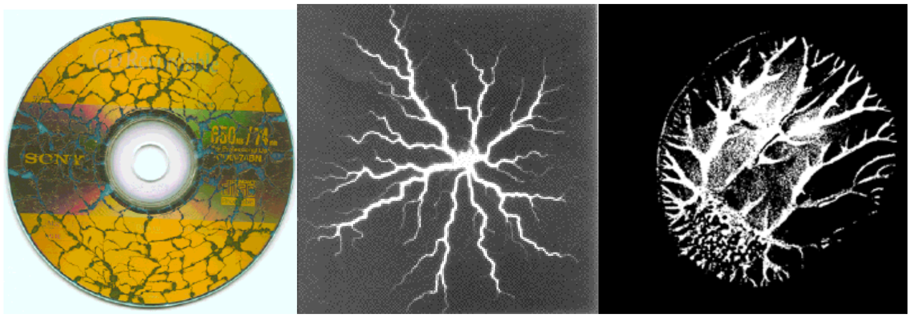
The students are tasked with making graphs of three different (x, y) relationships: a linear relationship, an exponential relationship, and a power law relationship. They discover that they can be differentiated because each one is a straight line on only one type of plot: a linear-linear, a log-linear, or a log-log plot. They also learn that each of these relationships has an interpretation. In a linear relationship, the change in y is proportional to the change in x, that is $\Delta y \propto \Delta x$. In an exponential relationship, the fractional change in y is proportional to the change in x, that is $\Delta y/y \propto \Delta x$. In a power law relationship, the fractional change in y is proportional to the fractional change in x, that is $\Delta y/y \propto \Delta x/x$.



Activity Type #4, MQR1, MQR 2, MQR 4, MQR 6, LO 1, LO 2, LO 5. Student projects identify fractals in nature, create them in the class, and present their results as oral presentations to the class and written reports.
Lecture 28: Newspaper Crumpling: Dimension, Density, and Scaling Relationships
Lecture29: Finding Fractals: Outside the Classroom
Lecture 30: Finding Fractals: Inside the Classroom
Objectives:

- We discover that fractals are all around us.
- We learn about the mechanisms that make these fractals.
- We can make fractals!
- We learn a about the mechanisms that make those fractals.

Students are tasked with finding fractals in books, on-line, and in the world around them. These can include: Lichtenberg figures (electric breakdown), growth around electrodes (chemical), or patterns around fracking wells (fluid dynamics). Students will also have the opportunity to make fractals from simple (inexpensive) items such as pulling apart pieces of plastic with toothpaste in between them, putting a CD-ROM (briefly and carefully) into a microwave oven or more complex projects if they are available in the Queens College Maker Space, such as injecting a low viscosity fluid into a high viscosity fluid held in the thin space between two flat plates (Hele Shaw cell).



Activity Type #5, MQR 1, MQR 2, MQR 3, MQR 5, **LO 2, LO 3, LO4, LO5**. Students will identify or measure dimensions of mathematical and physical objects and compare those quantitative results expected for those objects embedded in 1, 2, and 3 dimensions.
Lecture 14: The Middle Third Cantor Set: Making a Fractal by Removing Pieces
Lecture 15: The Sierpinski Triangle: Many Different Ways
Lecture 23: Many Different Dimensions: Fractal, Topological, Embedding
Lecture 24: Figuring Out the Fractal Dimension: Box Counting
Lecture 25: Figuring Out the Fractal Dimension: The Scaling Relationship Objectives:
Objectives:

- We learn the mathematical definitions of the fractal, topological, and embedding Dimensions
- We learn practical ways to measure these dimensions quantitatively
- We learn how to evaluate these quantitative measures for reasonableness compared to our expected estimations

The students are tasked with properly identifying the embedding dimension (space in which an object exists), the topological dimension (how the points of an object are connected) and the fractal dimension (how completely the object fills the embedding dimension). They will use box counting and scaling relationships to quantitatively measure the fractal dimensions of these objects and then compare the reasonableness of their results with their expectations based on the mathematical or physical properties of those objects.

Activity Type #6, MQR 1, MQR 2, MQR 5, MQR 6, LO 2, LO 5. Students will use what they have learned from the mathematical properties of fractals to help understand the properties and function of physical and biological systems.
Lecture 26: More Scaling Relationships: How Nerves Work
Lecture 27: Even More Scaling Relationships: The Mass and Density of Fractals
Lecture 33: Fractal Statistics: How the Ear Works and More
Lecture 34: Fractal Statistics: How the Heart Works and More
Objectives:

- We learn how nerves move information and the role that fractals play in how they work.
- We see how the amount of stuff depends on the size of an object.
- Some properties, that you might have thought were always the same, actually depend on the size of an object.
- We see how to use this to figure out the Fractal Dimension of an object.

The students are tasked with applying their mathematical knowledge of fractal properties to understand how those properties reflect or improve the structure and function of physical, chemical, biological, and medical systems. The natural systems studied include the scale and density of physical materials (in the design of wide frequency antennas and mechanical support structures in ice cream), the frequency of the shifts between protein conformations (in cell membrane ion channels), the information content in the timing of action potentials (in the ear), and the timing of heart beats (in patients with cardiac arrhythmias as measured by implanted cardioverter defibrillators).

Grading Policy
60% Group Work in Class
25% Exams: 10% Midterm Exam,15% Final Exam
15% Journal: answers to questions given with assignments

Policy on Missed or Late Assignments or Exams: There are no makeups for late assignments or missed exams, except for “exceptional circumstances” as determined by the instructor.

Students with Disabilities: Office of Special Services for Students with Disabilities/TRIO Frese Hall 111 | 718-997-5870. The Office of Special Services (OSS) ensures accessibility to all social and academic activities on campus. Students requiring accommodations must provide documentation identifying their disability. Once students register with OSS, they will receive accommodations and supportive services according to their disability needs in compliance with the American Disability Act of 1973. The intake process will include an academic plan to help students understand and set up their projected coursework. They are assigned an academic counselor and are monitored for their progress and success.

Policy on Academic Dishonesty: Academic dishonesty is prohibited in The City University of New York. Academic dishonesty includes: cheating, plagiarism, obtaining unfair advantage, and falsification of records and official documents. Penalties for academic dishonesty include academic sanctions, such as failing or otherwise reduced grades, and/or disciplinary sanctions, including suspension or expulsion. The full Queens College policy is available at: https://www.qc.cuny.edu/StudentLife/services/studev/Documents/Academic Integrity Violation Form RV.pdf

Policy on COVID-19: Course adjustments due to changing circumstances may require that this syllabus be updated at any time.

Course Calendar

Class Meeting	Day & Date	Topic	Reading/Assignment Lecture # from Mathematics and Science of Fractals by L. S. Liebovitch and L. A. Shehadeh, 2003	Objectives/Criteria Met
1	8/25/21	Introduction: Course Objectives	1	
2	8/30/21	Pre-Assessment: Students' Knowledge of Mathematics	2	LO 1
3	9/1/21	What is Mathematics?	3	LO 2, 5
4	9/6/21	A New Way to Count: The LOGARITHMIC Scale	4	LO 2, 5, MQR 1
5	9/8/21	More From Paper Folding: Logarithms	5	LO 5, MQR 2
6	9/13/21	Using Symbols to Represent Paper Folds: L's and R's	6	LO 3, MQR 2
7	9/15/21	Paper Folding Iterations: Real Folds in Real Paper and the Symbols L and R	7	LO 3, 4, MQR 3
8	9/20/21	Symmetry and Invariants: A More General Form of Self-Similarity	8	LO 2
9	9/22/21	Invariants: Examples from Science	9	LO 5, MQR 6
10	9/27/21	The Koch Curve: Introduction to the Snowflake	10	LO 2, MQR 2
11	9/29/21	The Koch Curve: Area and Limits	11	LO 2, MQR 2, 5
12	10/4/21	The Koch Curve: Zeno's Animals and The Area of the Koch Curve	12	MQR 2, 5
13	10/6/21	The Koch Curve: Length, Infinity, and Dimension	13	LO 4, MQR 2, 5
14	10/11/21	The Middle Third Cantor Set: Making a Fractal by Removing Pieces	14	LO2, 4, MQR 2, 5
15	10/13/21	The Sierpinski Triangle: Many Different Ways	15	LO 2, 3, MQR 3
16	10/18/21	Midterm Exam		
17	10/20/21	Paradigms: How The Eye Works	16	LO 5, MQR 6
18	10/25/21	Fractals in the Eye: Statistical Self-Similarity	17	LO 2, 5, MQR 1
19	10/27/21	What Graphs Tell Us: Discovering the Straight Lines	18	MQR 1, MQR 2
20	11/1/21	What Graphs Tell Us: What the Straight Lines Mean	19	MQR 1, MQR 2
21	11/3/21	Dimension: Exponents and Graphs	21	LO 4, MQR 1, 2
22	11/8/21	Dimension: Slopes and Logarithms	22	LO 4, MQR 1, 2
23	11/10/21	Many Different Dimensions: Fractal, Topological, Embedding	23	LO 2, 4, MQR 1, 2
24	11/15/21	Figuring Out the Fractal Dimension: Box Counting	24	LO2, 5, MQR 1, 2
25	11/17/21	Figuring Out the Fractal Dimension: The Scaling Relationship	25	LO 2, 5, MQR 1, 2
26	11/22/21	More Scaling Relationships: How Nerves Work	26	LO 5, MQR 1, 2, 5
27	11/24/21	Even More Scaling Relationships: The Mass and Density of Fractals	27	LO 2, MQR 1, 2
28	11/29/21	Newspaper Crumpling: Dimension, Density, and Scaling Relationships	28	LO 2, MQR 1, 2
29	12/1/21	Finding Fractals: Outside the Classroom	29	LO 1, 5, MQR 4, 6
30	12/6/21	Making Fractals: Inside the Classroom	30	LO 1, 5, MQR 4, 6
31	12/8/21	Making More Fractals: Inside the Classroom	31	LO 1, 5, MQR 4, 6
32	12/13/21	Review for Final Exam		