# Chap. 2. <u>The First Law</u> of Thermodynamics Law of Energy Conservation



System - part of the world we are interest in Surroundings - region outside the system World or Universe - system plus surroundings

Open system - transfer of matter between system and surroundings Closed system - no transfer of matter Isolated system - closed, no mechanical and thermal contact



First law holds however small the heat and work are.

Infinitesimal changes: dU = dw + dq

work on system due to expansion or contraction

$$dw = dw_{exp} + dw_e = -p_{ex}dV + dw_e$$

electric or other work on system



First law holds however small the heat and work are.

Infinitesimal changes: dU = dw + dq

work on system due to expansion or contraction

$$dw = dw_{exp} + dw_e = -p_{ex}dV + dw_e$$

electric or other work on system

Assume  $dw_e = 0$ . Then,  $dU = -p_{ex}dV + dq$ 

At constant volume,  $(dU)_V = (dq)_V = C_V dT$ 

 $C_V$  : Heat capacity at constant volume - the amount of heat transferred to the system per unit increase of temperature



First law holds however small the heat and work are.

Infinitesimal changes: dU = dw + dq

work due to expansion or contraction

$$dw = dw_{exp} + dw_e = -p_{ex}dV + dw_e$$

Assume  $dw_e = 0$ . Then,  $dU = -p_{ex}dV + dq$ 

At constant volume,  $(dU)_V = (dq)_V = C_V dT$ 

 $C_V$  : Heat capacity at constant volume - the amount of heat transferred to the system per unit increase of temperature

$$C_V = \left(\frac{dq}{dT}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$



Most processes occur at constant pressure. What is the relation between heat absorbed at constant pressure and the energy?

Enthalpy: H = U + pV

heat content, a state function in the unit of energy



The heat absorbed at constant pressure is the same as enthalpy change of the system given that its pressure is the same as the external pressure.

Enthalpy: H = U + pV

heat content, a state function in the unit of energy

$$dH = dU + Vdp + pdV = dq - p_{ex}dV + Vdp + pdV$$
  
 $dH = dq + Vdp + (p - p_{ex})dV$ 

If 
$$p = p_{ex}, dH = dq + Vdp$$

The heat absorbed at constant pressure is the same as enthalpy change of the system given that its pressure is the same as the external pressure.

Enthalpy: H = U + pV

heat content, a state function in the unit of energy

$$dH = dU + Vdp + pdV = dq - p_{ex}dV + Vdp + pdV$$
  
 $dH = dq + Vdp + (p - p_{ex})dV$ 

If 
$$p = p_{ex}, dH = dq + Vdp$$

At constant  $(dH)_p = (dq)_p = C_p dT$ pressure,

 $C_n$  : Heat capacity at constant pressure - the amount of heat

$$'p$$
 · transferred to the system per unit increase of temperature

$$C_p = \left(\frac{dq}{dT}\right)_p = \left(\frac{\partial H}{\partial T}\right)_p$$

# $C_V(T)$ and $C_p(T)$ are functions of temperature in general

**Molar Heat Capacities** 

$$C_{V,m}(T) = C_V(T)/n \qquad C_{p,m}(T) = C_p(T)/n$$



# $C_V(T)$ and $C_p(T)$ are functions of temperature in general

### **Molar Heat Capacities**

$$C_{V,m}(T) = C_V(T)/n \qquad \qquad C_{p,m}(T) = C_p(T)/n$$

#### Heat capacities can be used to determine U and H

Molar energy 
$$U_m(T_f) = U_m(T_i) + \int_{T_i,V}^{T_f} dT C_{V,m}(T)$$

Molar enthalpy 
$$H_m(T_f) = H_m(T_i) + \int_{T_i,p}^{T_f} dT C_{p,m}(T)$$





Example, Ideal Gas  $R = k_B N_A, k_B = 1.381 \times 10^{-23} \mathrm{JK}^{-1}$ 



 $C_V = \nu n R$ 





 $C_V = \nu n R$ 

 $H(T) = U(T) + pV = U(T) + nRT = H(0) + (\nu + 1)nRT$ 

Example, Ideal Gas  $R = k_B N_A, k_B = 1.381 \times 10^{-23} \text{JK}^{-1}$   $U(T) = U(0) + \nu n RT, \nu = \frac{3}{2}, \frac{5}{2}, \text{ or } 3 \longleftarrow \text{Nonlinear}$ Monoatomic 1 Inear molecule

 $C_V = \nu n R$ 

 $H(T) = U(T) + pV = U(T) + nRT = H(0) + (\nu + 1)nRT$ 

 $C_p = C_V + nR$ 

**Example, Ideal Gas**  $R = k_B N_A, k_B = 1.381 \times 10^{-23} \text{JK}^{-1}$  $U(T) = U(0) + \nu nRT, \nu = \frac{3}{2}, \frac{5}{2}, \text{ or } 3 \longleftarrow \text{Nonlinear molecule}$ Monoatomic 🖬 1 Linear molecule  $dU = C_V dT$  $C_V = \nu n R$  $w_{ad} = \Delta U = C_V (T_f - T_i)$  $H(T) = U(T) + pV = U(T) + nRT = H(0) + (\nu + 1)nRT$  $C_p = C_V + nR \qquad dH = C_p dT$ 

# **General Relations**

Consider U(T,V) as a function of T and V

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT = \pi_T dV + C_V dT$$



<u>A state function having the unit of pressure</u> (named as internal pressure in Atkins)

 $\pi_T = 0$  for ideal gas

#### **General Relations**

Consider U(T,V) as a function of T and V

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT = \pi_T dV + C_V dT$$



<u>A state function having the unit of pressure</u> (named as internal pressure in Atkins)

Then, consider U(T,p) and V(T,p) as functions of T and p  $\left(\frac{\partial U}{\partial T}\right)_p = \pi_T \left(\frac{\partial V}{\partial T}\right)_p + C_V = \alpha \pi_T V + C_V$  $\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p$ 

(isobaric) expansion coefficient

#### **General Relations**

Consider U(T,V) as a function of T and V

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT = \pi_T dV + C_V dT$$



<u>A state function having the unit of pressure</u> (named as internal pressure in Atkins)

Then, consider U(T,p) and V(T,p) as functions of T and p  $\left(\frac{\partial U}{\partial T}\right)_p = \pi_T \left(\frac{\partial V}{\partial T}\right)_p + C_V = \alpha \pi_T V + C_V$  $\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p$ 

(isobaric) expansion coefficient

$$C_p - C_V = \left(\frac{\partial H}{\partial T}\right)_p - C_V = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p - C_V = \alpha \pi_T V + p\alpha V = (\pi_T + p)\alpha V$$

(Note that H = U + pV)

**Another general relations** 

Consider temperature,  $T(H,p)\,$  , as a function of H and p

$$dT = \left(\frac{\partial T}{\partial H}\right)_p dH + \left(\frac{\partial T}{\partial p}\right)_H dp$$



**Another general relations** 

Consider temperature,  $T(H,p)\,$  , as a function of H and p

$$dT = \left(\frac{\partial T}{\partial H}\right)_p dH + \left(\frac{\partial T}{\partial p}\right)_H dp \tag{1}$$

Take partial derivative of (1) with respect to T while H remains fixed

$$\implies 1 = \left(\frac{\partial T}{\partial p}\right)_H \left(\frac{\partial p}{\partial T}\right)_H$$



(2)

**Another general relations** 

Consider temperature,  $T(H,p)\,$  , as a function of H and p

$$dT = \left(\frac{\partial T}{\partial H}\right)_p dH + \left(\frac{\partial T}{\partial p}\right)_H dp \tag{1}$$

Take partial derivative of (1) with respect to T while H remains fixed

$$\implies 1 = \left(\frac{\partial T}{\partial p}\right)_H \left(\frac{\partial p}{\partial T}\right)_H$$
(2)

Take partial derivative of (1) with respect to H while T remains fixed

$$\implies 0 = \left(\frac{\partial T}{\partial H}\right)_p \left(\frac{\partial H}{\partial p}\right)_T + \left(\frac{\partial T}{\partial p}\right)_H$$

$$(\frac{\partial H}{\partial p})_{T} = -\frac{\left(\frac{\partial T}{\partial p}\right)_{H}}{\left(\frac{\partial T}{\partial H}\right)_{p}} = -\left(\frac{\partial T}{\partial p}\right)_{H} \left(\frac{\partial H}{\partial T}\right)_{p} = -\mu \left(\frac{\partial H}{\partial T}\right)_{p}$$

$$(2) \qquad \mu = \left(\frac{\partial T}{\partial p}\right)_{H} \qquad \text{Joule-Thompson}$$



## Joule-Thompson Effect - Cooling of gas upon lowering of pressure at constant enthalpy (isenthalpic process)

 $\mu > 0$  for  $T_{I,low} < T < T_{I,up}$ 

Consider enthalpy,  $\,H(T,p)\,$ , as a function of T and p

$$dH = \left(\frac{\partial H}{\partial T}\right)_p dT + \left(\frac{\partial H}{\partial p}\right)_T dp = C_p dT - \mu C_p dp$$

Isothermal Joule-Thompson coefficient

$$\mu_T = \left(\frac{\partial H}{\partial p}\right)_T = -\mu C_p$$

**Easier to measure** 





<u>Hess' s Law</u>

The standard enthalpy of an overall reaction is the sum of the standard enthalpies of the individual reactions into which a reaction may be divided.

Direct result of enthalpy as a state function.

If 
$$A \to B, \Delta_{A \to B} H^{\ominus}$$
 and  $B \to C, \Delta_{B \to C} H^{\ominus}$   
then  $\Delta_{A \to C} H^{\ominus} = \Delta_{A \to B} H^{\ominus} + \Delta_{B \to C} H^{\ominus}$ 

### <u>Kirchhoff's law</u>

$$\begin{split} \Delta_r H^{\ominus}(T_2) &= \Delta_r H^{\ominus}(T_1) + \int_{T_1}^{T_2} dT \Delta_r C_p^{\ominus} & \text{at constant pressure} \\ \hline \text{Reaction heat capacity} & \Delta_r C_p^{\ominus} &= \sum_f \nu(f) C_{p,m}^{\ominus}(f) - \sum_i \nu(i) C_{p,m}^{\ominus}(i) \\ & \text{for the chemical reaction} & \sum_i \nu(i) M_i \to \sum_f \nu(f) M_f \end{split}$$

