Chapter 2. Review of Quantum Mechanics

Assume that there is a quantum mechanical state. Then, there is a unique ket $|\cdots\rangle$ or equivalently a bra $\langle\cdots|$

There is an infinite number of ways to create a quantum mechanical state.

For two quantum mechanical states $|\alpha\rangle$ and $|\beta\rangle$

$|\gamma\rangle = C_1|\alpha\rangle + C_2|\beta\rangle$ corresponds to another quantum mechanical state.

Inner product: $\langle\alpha|\beta\rangle$ - complex number, $\langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$

Outer product: $|\alpha\rangle\langle\beta|$- operator, $(c|\alpha\rangle\langle\beta|)^\dagger = c^*|\beta\rangle\langle\alpha|$

$$(|\alpha\rangle\langle\beta|)(|\gamma\rangle\langle\delta|) = |\alpha\rangle\langle\beta|\gamma\rangle\langle\delta| = (\langle\beta|\gamma\rangle)|\alpha\rangle\langle\delta|$$

$$(|\alpha\rangle\langle\beta|\gamma\rangle\langle\delta|)^\dagger = \langle\beta|\gamma\rangle^*|\delta\rangle\langle\alpha| = \langle\gamma|\beta\rangle|\delta\rangle\langle\alpha| = |\delta\rangle\langle\gamma|\beta\rangle\langle\alpha|$$

Any operator can be made Hermitian by adding its Hermitian conjugate

$$|\alpha\rangle\langle\beta| + (|\alpha\rangle\langle\beta|)^\dagger = |\alpha\rangle\langle\beta| + |\beta\rangle\langle\alpha|$$

For any Hermitian operator $\hat{A}$

$e^{i\hat{A}}$ is an unitary operator.

Position and momentum operators are Hermitian and satisfy the following fundamental relation: $[\hat{x},\hat{p}] = i\hbar$

From the above commutator identity, one can show that

$$\hat{p}|x\rangle = -\frac{\hbar}{i}\frac{\partial}{\partial x}|x\rangle \qquad \text{or} \qquad \langle x|\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}\langle x|$$

Mathematical operation

Quantum mechanical operation

Commutator relations define properties of operators and it is important to understand simple algebra involving commutators

$$[\hat{A}+\hat{B},\hat{C}] = [\hat{A},\hat{C}] + [\hat{B},\hat{C}]$$

$$[\hat{A},\hat{B}] = -[\hat{B},\hat{A}]$$

$$[c\hat{A},\hat{B}] = c[\hat{A},\hat{B}]$$

$$[\hat{A}\hat{B},\hat{C}] = \hat{A}[\hat{B},\hat{C}] + [\hat{A},\hat{C}]\hat{B}$$

BCH identity:

$$e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A},\hat{B}] + \frac{1}{2}[\hat{A},[\hat{A},\hat{B}]] + \cdots + \frac{1}{n!}[\hat{A},\cdots,[\hat{A},B]\cdots] + \ldots$$

Dimension can be extended by direct product - juxtaposition of all the degrees of freedom

Direct product is well defined between any well-defined independent degrees of freedom. Examples are all the cartesian coordinates of different atoms.

Schrodinger's equation in Dirac notation

$$\hat{H}|E\rangle = \left(\frac{\hat{p}^2}{2m} + V(\hat{x})\right)|E\rangle = E|E\rangle$$

$$i\hbar\frac{\partial}{\partial t}|\alpha;t\rangle = \hat{H}|\alpha;t\rangle = \left(\frac{\hat{p}^2}{2m} + V(\hat{x})\right)|\alpha;t\rangle$$

$$|\alpha;t\rangle = e^{-i\hat{H}t/\hbar}|\alpha;0\rangle$$