Chapter 3. Matter radiation interaction

Radiation is an electromagnetic field satisfying the following set of Maxwell's equations:

In vacuum

$$\nabla \cdot \mathbf{E} = 4\pi\rho \ ,$$
$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{J} \ ,$$
$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0 \ ,$$
$$\nabla \cdot \mathbf{B} = 0$$

In medium

$$\nabla \cdot \mathbf{D} = 4\pi\rho \ ,$$
$$\nabla \times \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = \frac{4\pi}{c}\mathbf{J} \ ,$$
$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0 \ ,$$
$$\nabla \cdot \mathbf{B} = 0$$

For Linear medium,

$$\mathbf{D} = \epsilon\mathbf{E}$$
$$\mathbf{B} = \mu\mathbf{H}$$

$\longrightarrow$

$$\nabla \cdot \mathbf{E} = \frac{4\pi\rho}{\epsilon}$$
$$\nabla \times \mathbf{B} - \frac{\epsilon\mu}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi\mu}{c}\mathbf{J}$$

Magnetic Field can be considered as the curl of Vector Potential

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$$

Coulomb gauage $\boldsymbol{\nabla} \cdot \mathbf{A} = 0$ and free space

$$\nabla^2 \mathbf{A} - \frac{\epsilon\mu}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = 0$$

The resulting electromagnetic waves are the following plane waves

$$\begin{aligned} \mathbf{B}(\mathbf{r},t) &= A_0(\mathbf{k} \times \mathbf{u}_e)\sin(\omega t - \mathbf{k}\cdot\mathbf{r}) = A_0 k \mathbf{u}_b \sin(\omega t - \mathbf{k}\cdot\mathbf{r}) \\ \mathbf{E}(\mathbf{r},t) &= \frac{A_0\omega}{c}\mathbf{u}_e \sin(\omega t - \mathbf{k}\cdot\mathbf{r}) = \frac{A_0 k}{n}\mathbf{u}_e \sin(\omega t - \mathbf{k}\cdot\mathbf{r}) \end{aligned}$$

Force applied to a classical particle of charge q

$$m\frac{d^2\mathbf{r}}{dt^2} = q\left(\mathbf{E}(\mathbf{r},t) + \frac{1}{c}\frac{d\mathbf{r}}{dt}\times\mathbf{B}(\mathbf{r},t)\right) - \boldsymbol{\nabla}V(\mathbf{r})$$

Classical Hamiltonian: $$H = \frac{1}{2m}\left(\mathbf{p} - \frac{q}{c}\mathbf{A}(\mathbf{r},t)\right)^2 + q\Phi(\mathbf{r},t) + V(\mathbf{r})$$

Quantum Hamiltonian: $$\hat{H} = \frac{1}{2m}\left(\hat{\mathbf{p}} - \frac{q}{c}\mathbf{A}(\hat{\mathbf{r}},t)\right)^2 + q\Phi(\hat{\mathbf{r}},t) + V(\hat{\mathbf{r}})$$

In the weak field limit,

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_1(t)$$

$$\hat{H}_1(t) = q\Phi(\hat{\mathbf{r}},t) - \frac{q}{mc}\mathbf{A}(\hat{\mathbf{r}},t)\cdot\hat{\mathbf{p}}$$

For monochromatic radiation,

$$\Phi = 0$$

$$\mathbf{A} = A_0 \mathbf{u}_e \cos(\mathbf{k}\cdot\mathbf{r} - \omega t)$$

$$\longrightarrow \quad \hat{H}_1(t) = -\frac{q}{mc} A_0 \left(e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \right) \mathbf{u}_e \cdot \hat{\mathbf{p}}$$

Molecular length scale is much smaller than the wavelength of radiation in most cases.

$$\mathbf{k}\cdot\hat{\mathbf{r}} \approx 0 \quad \text{(Dipole approximation)}$$

$$\begin{aligned} \hat{H}_1(t) &= -\frac{qA_0}{2mc} \left(e^{-i\omega t} + e^{i\omega t} \right) \mathbf{u}_e \cdot \hat{\mathbf{p}} \\ &= -\frac{iA_0}{2\hbar c} \left(e^{-i\omega t} + e^{i\omega t} \right) [\hat{H}_0, \mathbf{u}_e \cdot \hat{q}\mathbf{r}] \end{aligned}$$