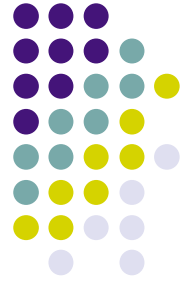


Chapter 3. Matter radiation interaction



Radiation is an electromagnetic field satisfying the following set of Maxwell's equations:

In vacuum

$$\begin{aligned}\nabla \cdot \mathbf{E} &= 4\pi\rho , \\ \nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} &= \frac{4\pi}{c} \mathbf{J} , \\ \nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} &= 0 , \\ \nabla \cdot \mathbf{B} &= 0\end{aligned}$$

In medium

$$\begin{aligned}\nabla \cdot \mathbf{D} &= 4\pi\rho , \\ \nabla \times \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} &= \frac{4\pi}{c} \mathbf{J} , \\ \nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} &= 0 , \\ \nabla \cdot \mathbf{B} &= 0\end{aligned}$$

For Linear medium,

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$



$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{4\pi\rho}{\epsilon} \\ \nabla \times \mathbf{B} - \frac{\epsilon\mu}{c} \frac{\partial \mathbf{E}}{\partial t} &= \frac{4\pi\mu}{c} \mathbf{J}\end{aligned}$$

Magnetic Field can be considered as the curl of Vector Potential

$$\mathbf{B} = \nabla \times \mathbf{A}$$

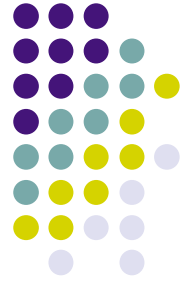
Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ and free space

$$\nabla^2 \mathbf{A} - \frac{\epsilon\mu}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = 0$$

The resulting electromagnetic waves are the following plane waves

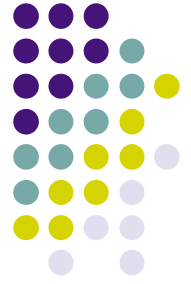
$$\mathbf{B}(\mathbf{r}, t) = A_0(\mathbf{k} \times \mathbf{u}_e) \sin(\omega t - \mathbf{k} \cdot \mathbf{r}) = A_0 k \mathbf{u}_b \sin(\omega t - \mathbf{k} \cdot \mathbf{r})$$

$$\mathbf{E}(\mathbf{r}, t) = \frac{A_0 \omega}{c} \mathbf{u}_e \sin(\omega t - \mathbf{k} \cdot \mathbf{r}) = \frac{A_0 k}{n} \mathbf{u}_e \sin(\omega t - \mathbf{k} \cdot \mathbf{r})$$



Force applied to a classical particle of charge q

$$m \frac{d^2 \mathbf{r}}{dt^2} = q \left(\mathbf{E}(\mathbf{r}, t) + \frac{1}{c} \frac{d\mathbf{r}}{dt} \times \mathbf{B}(\mathbf{r}, t) \right) - \nabla V(\mathbf{r})$$



Classical Hamiltonian: $H = \frac{1}{2m} \left(\mathbf{p} - \frac{q}{c} \mathbf{A}(\mathbf{r}, t) \right)^2 + q\Phi(\mathbf{r}, t) + V(\mathbf{r})$

Quantum Hamiltonian: $\hat{H} = \frac{1}{2m} \left(\hat{\mathbf{p}} - \frac{q}{c} \mathbf{A}(\hat{\mathbf{r}}, t) \right)^2 + q\Phi(\hat{\mathbf{r}}, t) + V(\hat{\mathbf{r}})$

In the weak field limit,

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_1(t)$$

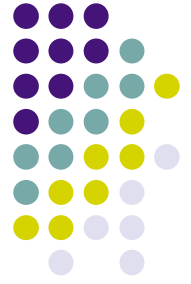
$$\hat{H}_1(t) = q\Phi(\hat{\mathbf{r}}, t) - \frac{q}{mc} \mathbf{A}(\hat{\mathbf{r}}, t) \cdot \hat{\mathbf{p}}$$

For monochromatic radiation,

$$\Phi = 0$$

$$\mathbf{A} = A_0 \mathbf{u}_e \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$$

$$\rightarrow \hat{H}_1(t) = -\frac{q}{mc} A_0 \left(e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} + e^{-i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \right) \mathbf{u}_e \cdot \hat{\mathbf{p}}$$



Molecular length scale is much smaller than the wavelength of radiation in most cases.

$$\mathbf{k} \cdot \hat{\mathbf{r}} \approx 0 \quad (\text{Dipole approximation})$$

$$\begin{aligned} \hat{H}_1(t) &= -\frac{qA_0}{2mc} (e^{-i\omega t} + e^{i\omega t}) \mathbf{u}_e \cdot \hat{\mathbf{p}} \\ &= -\frac{iA_0}{2\hbar c} (e^{-i\omega t} + e^{i\omega t}) [\hat{H}_0, \mathbf{u}_e \cdot \hat{\mathbf{q}}\mathbf{r}] \end{aligned}$$