QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 122 **SPRING 2025**

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total
8	12	12	8	8	10	12	8	12	10	100

Table: Point Values for each question (Total 100)

Instructions: Answer all questions. Show all work. Note that calculators are permitted only for specified questions.

1. (8 points) Consider the quadratic function $p(x) = -3x^2 + 12x - 9$.
   a) Rewrite $p(x)$ in standard vertex form by completing the square.
   b) Find the coordinates of the vertex.
   c) Use algebra to find the x- and y-intercepts.
   d) Sketch the graph $y = p(x)$. Label the vertex and all intercepts.
   e) State the maximum or minimum value of $p(x)$.
   f) Solve the inequality $p(x) > 0$ algebraically and express the solution in interval notation.

2. (12 points) Solve for x in each of the following. Show all steps.
   a) $2^{3x-1} = 16^{x+2}$
   b) $\log_3(x-10) + \log_3(x-2) = 2$
   c) $e^{2x} - 7e^x + 12 = 0$ **(Hint: Use a substitution.)**
   d) $\log_2(\log_4(x)) = 1$

3. (12 points) Given the polynomial $P(x) = x^4 - 5x^3 + 4x^2$.
   a) Factor $P(x)$ completely and find all zeros, stating their multiplicities.
   b) Describe the end behavior of $P(x)$ as $x \to \infty$ and $x \to -\infty$.
   c) Sketch the graph of $y = P(x)$, labeling all intercepts.

4. (8 points) Find the exact value of each expression without a calculator. Show all work.
   a) $\sin(\tan^{-1}(2))$
   b) $2\cos^2\left(\frac{5\pi}{12}\right) - 1$
   c) $\csc\left(\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)\right)$
   d) $\log_5(50) - \log_5(2) + \ln(e^3)$

5. (8 points) Verify the given identity. Show all steps clearly.

$$\frac{\sin x}{1 - \cos x} - \frac{\cos x}{\sin x} = \csc x$$

6. (10 points) Sketch the graph of each function using transformations of a parent function. Label all intercepts and asymptotes where applicable.
   a) $y = -\sqrt{x-4} + 2$
   b) $y = -3e^{x+1} + 3$
   c) $y = 2\sin\left(x - \frac{\pi}{2}\right)$ (Sketch one complete period. State the amplitude and period.)

(continued on the back)

7. (12 points) Let $f(x) = \frac{2x}{9-x}$ and $g(x) = 5 - \sqrt{x}$.

a) Determine the domain of f and the domain of g in interval notation.
b) Use algebra to determine the equations of the vertical and horizontal asymptotes of $y = f(x)$.
c) Sketch the graph of $y = f(x)$, labeling all intercepts and asymptotes.
d) Determine the range of f.
e) Find $f^{-1}(x)$ algebraically.
f) State the domain and range of $f^{-1}(x)$.
g) Determine $(g \circ f)(x)$. Don't simplify.
h) Evaluate and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$, where $h = 1$.

8. (8 points) A child is flying two kites at the same time, with 10 meters of line out to one kite and 20 meters let out to the other. We estimate the angle between the two lines to be $60°$. What is the distance between the two kites? **(You are welcome to use your calculator.)**
(Hint: Consider using either the Law of Sines or Law of Cosines.)

9. (12 points) Express all answers in exact form. Given $\sin A = \frac{5}{13}$ where A is in Quadrant II, and $\cos B = -\frac{8}{17}$ where B is in Quadrant III, find:

a) $\sin(A - B)$
b) $\cos(2A)$
c) $\tan(A) - \tan(B)$

10. (10 points) A man invests \$ 5,000 in an account that pays 6% interest per year, compounded quarterly. **(You are welcome to use your calculator.)**

a) Find the amount after 5 years. Round answer to two decimal places.
b) How long will it take for the investment to grow to \$ 11,000? Round your answer to one decimal place.