QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 141 Spring 2025

Instructions: Show your work except in Question 1. Phones must be put away.

1. Draw the graph of a function which has all of the following properties:

   (a) $\lim_{x\to 3^+} f(x) = 2$
   (b) $\lim_{x\to 3^-} f(x) = 3$
   (c) $f(3) = 6$
   (d) $\lim_{x\to\infty} f(x) = 3$
   (e) $\lim_{x\to-\infty} f(x) = 0$
   (f) $\lim_{x\to 5} f(x) = \infty$

2. Compute each of the following limits without a calculator. If a limit is ∞ or $-\infty$, say so and justify your conclusion.

   (a) $\lim_{x\to 2} \dfrac{\sqrt{x^2+5}-3}{x-2}$
   (b) $\lim_{x\to 5} \dfrac{x^2-6x+5}{x^2-25}$
   (c) $\lim_{x\to 3^-} \dfrac{5x-1}{(x-3)(x+1)}$
   (d) $\lim_{x\to 0} \dfrac{(\sin 3x)(\sin 4x)}{x^2}$

3. Use your calculator to make a table to estimate the following limit, accurate to 3 decimal places:

$$\lim_{x\to 0} \frac{2^x - 3^x}{5^x - 1}$$

   Copy your table to the booklet and state what your estimate is for the limit.

4. Compute $f'(x)$ using the DEFINITION of the derivative if $f(x) = \dfrac{5}{x} + 3$.

5. Compute y' (or $\dfrac{dy}{dx}$) for each of the following (Algebraic simplification is not necessary.):

   (a) $y = \dfrac{4}{x^2} + 3\sqrt{x} + \sqrt[3]{3}$
   (b) $y = \dfrac{\sin(3x^3)}{\tan(2x)}$
   (c) $y = (2x^2+3)^2(\sec(10x))$
   (d) $y = \sin(\cos x)$
   (e) $4x^2y + 3y^3 = 8x$

6. Using calculus, find the absolute maximum and minimum of $f(x) = x^3 - 3x^2 + 7$ on $[1, 3]$.

(continued on the back)

7. A spherical balloon of radius r and volume $V = \frac{4}{3}\pi r^3$ is being heated and is expanding. When the radius is 3 inches, the volume is increasing at the rate of 4 cubic inches per second. Find the exact rate at which the radius is increasing at that instant.

8. An object is moving along a straight line and its position, $s(t)$, at any time $0 \leq t \leq \pi/4$ is given by $s(t) = 4\tan t$. Find expressions for the velocity and acceleration of the object during this time interval.

9. There are many rectangles whose area is 225. Using calculus, find the dimensions of these rectangles which has the smallest perimeter. Be sure you tell how you know the perimeter is actually a minimum.

10. Let $f(x) = \dfrac{x}{x-2}$.
    (a) Using calculus, find equations of the vertical and horizontal asymptotes.
    (b) Using calculus, find the intervals of increase and decrease.
    (c) Using calculus, find the intervals of concavity.
    (d) Sketch the graph of f based on your answers to parts (a)-(c).