



Department of Mathematics
Math 152 Final Exam, Spring 2025

- The exam has two parts.
- You have 150 minutes to answer the questions.
- Show all work in your exam book.

Part I [30 points]. Write the letter of the correct answer in your exam book. Show your work.

1. An experimental drug decays at the relative rate of 0.095 per hour. How many hours does it take for the quantity present in the bloodstream to drop to 20% of its initial value?

(A) 19 (B) 17 (C) 15 (D) 13

2. $\lim_{x \rightarrow 0} \frac{\sin(2x) - 2 \sin x}{x^3}$ is

(A) -1 (B) -3 (C) -9 (D) $-\infty$

3. The integral $\int_0^3 \frac{1}{x-2} dx$ is

(A) 0 (B) $\ln 2$ (C) $-\ln 2$ (D) divergent

4. I set up an integral for the arc length of $y = \ln x$ between $x = 1$ and $x = 4$. My calculator estimates this integral to be

(A) 3.3243 (B) 3.3428 (C) 3.3657 (D) 3.3803

5. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{3^n}{n} (x+1)^n$ is

(A) $\left(-\frac{4}{3}, -\frac{2}{3}\right)$ (B) $\left[-\frac{4}{3}, -\frac{2}{3}\right]$ (C) $\left[-\frac{4}{3}, -\frac{2}{3}\right)$ (D) $\left(-\frac{4}{3}, -\frac{2}{3}\right]$

6. Ignoring the constant of integration, the power series of $\int \frac{x}{1+x^3} dx$ centered at 0 has the form

(A) $x - \frac{x^4}{4} + \frac{x^7}{7} - \frac{x^{10}}{10} + \dots$ (B) $x + \frac{x^4}{4} + \frac{x^7}{7} + \frac{x^{10}}{10} + \dots$
(C) $\frac{x^2}{2} - \frac{x^5}{5} + \frac{x^8}{8} - \frac{x^{11}}{11} + \dots$ (D) $\frac{x^2}{2} + \frac{x^5}{5} + \frac{x^8}{8} + \frac{x^{11}}{11} + \dots$

(continued on the other side)

Part II [70 points]. Solve the following six problems.

Problem 1. [12 points] In each case, find the derivative $y' = \frac{dy}{dx}$:

- (a) $y = \ln(e^{3x} + e^{-3x})$
- (b) $y = \tan^{-1}(\sqrt{x})$
- (c) $y = (1 - 2x)^{\sin x}$ (Hint: Use logarithmic differentiation.)

Problem 2. [15 points] Evaluate the following integrals:

- (a) $\int \sqrt{16 - x^2} \, dx$
- (b) $\int \frac{5x^2 - 2x + 1}{x^3 - x^2} \, dx$
- (c) $\int_3^\infty \frac{1}{x(\ln x)^2} \, dx$

Problem 3. [10 points] Let R be the region in the plane bounded by the curve $y = e^x$ and the x -axis, between $x = 0$ and $x = 1$.

- (a) Find the volume of the solid obtained by rotating R around the horizontal line $y = -1$.
- (b) Find the volume of the solid obtained by rotating R around the y -axis.

Problem 4. [8 points] Find the explicit form of the solution $y = y(x)$ of the differential equation

$$\frac{dy}{dx} = (1 + x)(1 + y^2)$$

which satisfies the condition $y(0) = 1$.

Problem 5. [15 points] Using appropriate tests, determine the convergence or divergence of each of the following series:

- (a) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{3n^2 - 1}$
- (b) $\sum_{n=1}^{\infty} \frac{n\sqrt{n}}{n^3 + 5}$
- (c) $\sum_{n=1}^{\infty} \frac{n^n}{(-3)^n n!}$

Problem 6. [10 points]

- (a) Using your knowledge of the Maclaurin series of the sine function, find the Maclaurin series of $f(x) = \sin(x^2)$.
- (b) Use your answer to (a) to write the definite integral

$$I = \int_0^1 \sin(x^2) \, dx$$

as an alternating series. Then estimate the value of I with an error of less than 0.0001.