QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2.5 Hours

Mathematics 152 **Fall 2025**

Instructions: **Answer all questions.** **Show all work.**

1. Compute the following limits. (Both ∞ and $-\infty$ are acceptable answers.)
   (a) (5 points) $\lim_{x\to 0^+} (1+\tan x)^{\frac{1}{x}}$
   (b) (5 points) $\lim_{x\to 0} \dfrac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$

2. Compute $\dfrac{dy}{dx}$ for the following functions.
   (a) (5 points) $y = (4 + 2e^{3x})^5$
   (b) (5 points) $y = \ln\left(\dfrac{(4x+1)^3}{\sqrt{x^2+1}}\right)$

3. Integrate the following:
   (a) (6 points) $\displaystyle\int \sin^3 x \cos^2 x \, dx$
   (b) (6 points) $\displaystyle\int \frac{x^2+1}{x(x+1)(x-1)}\, dx$
   (c) (6 points) $\displaystyle\int \frac{1}{(4-x^2)^{\frac{3}{2}}}\, dx$
   (d) (6 points) $\displaystyle\int \frac{\ln x}{\sqrt{x}}\, dx$

4. (4 points) Determine whether the improper integral $\displaystyle\int_{-\infty}^{0} \frac{dx}{7-2x}$ is convergent or divergent, and if it is convergent, compute its value.

5. Let R be the region bounded by $y = x^2$, $x = 2$, and $y = 0$.
   (a) (2 points) Find the area of R.
   (b) (5 points) Find the volume of the solid obtained by rotating the region R around the y-axis, using the method of *cylindrical shells*.
   (c) (5 points) Compute the same volume, but this time using the method of *washers*.

6. (5 points) Set up, but DO NOT EVALUATE, an integral expression for the arc length of the curve $y = \sec x$ between the point $(0, 1)$ and $(\frac{\pi}{3}, 2)$.

7. (5 points) Solve the differential equation $e^x \dfrac{dy}{dx} = y$ with the condition $y(0) = 1$.

8. Determine whether the following infinite series converge or diverge, and justify your answers by applying an appropriate convergence test:
   (a) (4 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{4n^5-1}}$
   (b) (4 points) $\displaystyle\sum_{n=1}^{\infty} \frac{100^n}{n^n}$
   (c) (4 points) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln(n+1)}$

9. (4 points) Find the radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-7)^n}{n^5} x^n$

10. (4 points) Find the Maclaurin series of the function $f(x) = \dfrac{2}{1-x^7}$. Your answer should be an infinite series written in summation notation. (Hint: Rather than trying to compute this directly, start with a more well-known Maclaurin series.) Also find its interval of convergence.

11. Let $f(x) = \displaystyle\int_e^x (\ln t)^{-2}\, dt$ for all $x > 1$.
    (a) (5 points) Calculate the **second** degree Taylor polynomial of $f(x)$ centered at e.
    (b) (5 points) Use Taylor's Formula to find an upper bound on $|R_1(3)|$, which denotes the error in computing $f(3)$ using the **first** degree Taylor polynomial of $f(x)$ centered at e.