

Name: _____ CUNY ID: _____

Spring 2026: Math 122 Uniform Final Exam

Instructions:

- Calculators are **not** permitted for use.
- Maximum credit is only assigned to well-written solutions that **demonstrate** mathematical proficiency and conceptual understanding. Illegible work will be ignored.
- There are questions on the front **and** back of each page.

1. Let $p(x) = 25x^2 - 19$ and $g(x) = \sqrt{x + 3}$

a) Evaluate and simplify $p(-2x) + g(33)$. (3 points)

b) Find $(g \circ p)(x)$. (2 points)

c) State the domain of $(g \circ p)(x)$ using interval notation. (4 points)

2. Consider the function $g(x) = -x^2 + 2x - 5$.

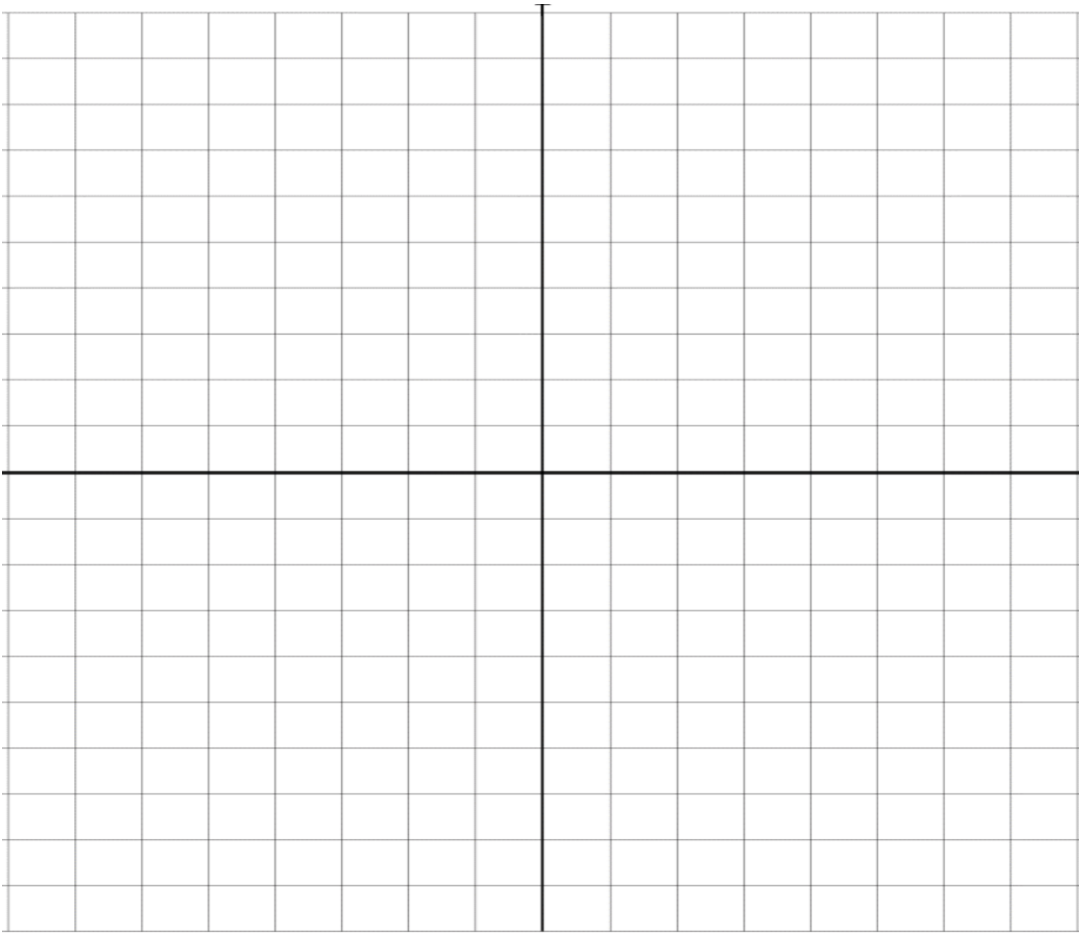
a) Use completing the square or any other algebraic method to express $g(x)$ in vertex form. (3 points)

b) State the coordinates of the vertex of $g(x)$. (1 point)

c) Use algebra to find any x - and y - intercepts of $g(x)$. If there are none, write NONE. (3 points)

2. (Continued)

- d)** Sketch $g(x)$ on the provided set of axes below. Make sure to **label** the vertex and any intercepts.
(3 points)



- e)** State the intervals of increase and decrease for $g(x)$ in interval notation. (3 points)
- f)** Use relevant facts about functions to explain why $g(x)$ does not have an inverse function on its entire domain. (2 points)

3. Consider the one-to-one function $f(x) = \frac{2x}{3-x}$.

a) Find any vertical and horizontal asymptotes of $f(x)$ by their equation. (2 points)

b) State the domain of $f(x)$ in interval notation. (1 point)

c) Use algebra to find the inverse function, $f^{-1}(x)$. (3 points)

d) State the range of $f^{-1}(x)$ interval notation. (1 point)

4. Consider a polynomial function $p(x)$ with the following zeros and respective multiplicities:

Zero	Multiplicity
-5	1
-4	1
1	3
2	2

- a) State the degree of $p(x)$. (2 points)
- b) If the y -intercept is $(0,160)$, write an equation for $p(x)$ in factored form. (3 points)
- c) Describe the end-behavior of $p(x)$ based on your work in **a)** and **b)**. (2 points)

5. The parts for this question are **unrelated**.

- a) Let $f(x) = \frac{2}{x+4}$. Use algebra to find and simplify the difference quotient, $\frac{f(x+h)-f(x)}{h}$, where $h \neq 0$.
(4 points)

- b) Using your knowledge of trigonometry, evaluate the following expression. (3 points)

$$\cot\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$$

6. Use algebra to find all solutions for the following equations.

a) $3xe^x + x^2e^x = 0$ (3 points)

b) $\log_{15}(x + 7) + \log_{15}(x - 7) = 1$ (4 points)

c) $4 \sin(x) \cos(x) - 2 \sin(x) + 2 \cos(x) - 1 = 0$ on the interval $[0, 2\pi)$ (4 points)

7. Consider the following piecewise defined function.

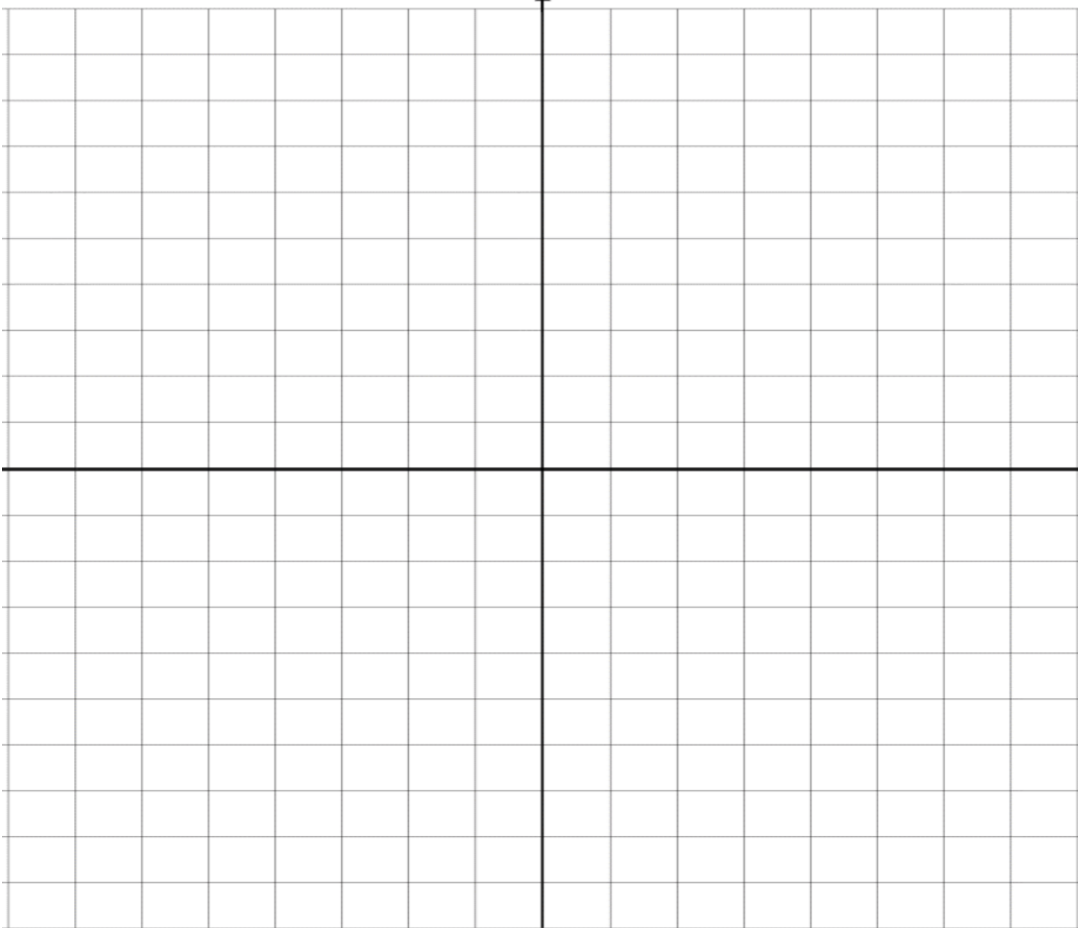
$$g(x) = \begin{cases} 2^{x+1} - 1 & \text{if } x < 1 \\ |x - 3| + 1 & \text{if } x > 1 \end{cases}$$

a) Find any asymptotes by their equation of $g(x)$. (2 points)

b) Use algebra to find all x - and y - intercepts of $g(x)$. Leave your answers as exact numbers where applicable. (3 points)

7. (Continued)

- c) Sketch the graph of $g(x)$ either by using transformations or by making a table of values. (6 points)
- **Plot and label** the coordinates for any/all intercepts.
 - **Plot and label** the coordinates for any/all holes.
 - **Plot and label** the coordinates for any/all vertices.
 - **Sketch and label** any asymptotes by their equation.



- d) State the range of $g(x)$ in interval notation. (2 points)

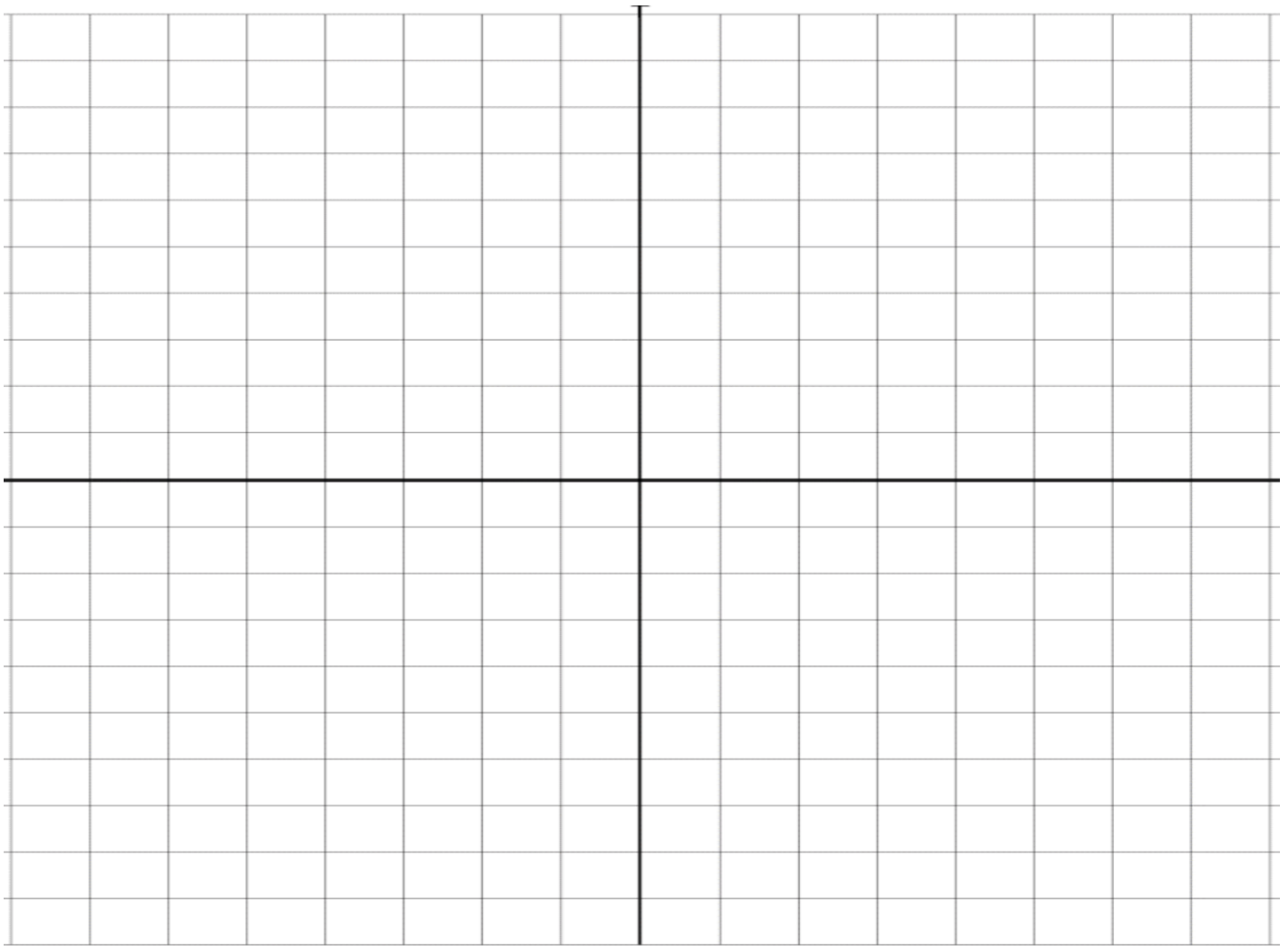
8. Let $F(x) = 6x^4 - 13x^3 - 8x^2 + 17x + 6$

a) Use the remainder theorem to show that $x = 2$ is a zero of $F(x)$. (2 points)

b) Use algebra and applicable facts and theorems of polynomials to write $F(x)$ in fully factored form. (4 points)

8. (Continued)

- c) Sketch a reasonably accurate graph of $F(x)$ on the provided set of axes below. Make sure to **label** all x - and y - intercepts. (6 points)



9. Use algebra and your knowledge of trigonometric expressions to prove the following identity: (3 points)

$$\cos(x) + 1 = \frac{\sin^2(x)}{1 - \cos(x)}$$

- 10.** Let A be an angle in Quadrant II such that $\sin(A) = \frac{\sqrt{3}}{2}$, and let B be an angle in Quadrant III such that $\cos(B) = -\frac{\sqrt{2}}{2}$. Using your knowledge of trigonometric expressions, evaluate each of the following:
- a)** $\sin(2A)$ (2 points)

b) $\cos(2B)$ (2 points)

c) $\cos(A - B)$ (3 points)

11. Consider the function $f(x) = \sin(x)$

a) Let $g(x)$ be the graph of $f(x)$ with a phase shift of $\frac{\pi}{2}$ units to the left, a vertical shift upwards by 1 unit, and a reflection across the midline.

- Write an equation for $g(x)$. (3 points)

b) Sketch the graph of $g(x)$ on the interval $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$. Your sketch should have at least 5 clearly labeled points. (6 points)

