

**QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS**

Final Examination

$2\frac{1}{2}$ Hours

Mathematics 131

Spring 2026

Instructions: Answer all questions. Show all work.

1. Evaluate each of the following limits without the use of a table or graph.

a) $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 + x - 6}$

b) $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{9 - x}$

c) $\lim_{x \rightarrow -\infty} \frac{5x^2 - 6x + 8}{7x^2 + 9}$

d) $\lim_{x \rightarrow \infty} \frac{-5x^4 + 3x^2 - 1}{x^3 + 4x^2 + 6x}$

e) $\lim_{x \rightarrow 3} f(x)$, given $f(x) = \begin{cases} 5 - x & x < 3 \\ 6 & x = 3 \\ x^2 - 7 & x > 3 \end{cases}$

2. Find $\frac{dy}{dx}$ for each of the following. (You do not need to simplify.)

a) $y = \frac{x^4}{5} + \frac{7}{x^3} - \frac{x^5}{x^2} + \left(\frac{5}{x^4}\right)^3 - \sqrt{e}$

b) $y = \frac{\sqrt{4x^3 - x + 5}}{e^{5x} + e^{-x}}$

c) $y = \ln\left(\frac{e^x}{5x^4 - x^3 + 2x}\right)$

d) $e^{2y} + 5x^2y^4 - x = 3y + 7$

e) $y = \left(\sqrt[3]{2x^3 - 5x}\right)^{\ln x}$ (Hint: use logarithmic differentiation)

3. Let $f(x) = -x^2 + 5x - 1$

- a) Use the limit definition of the derivative to find $f'(x)$.
- b) Find the slope of the tangent line of $f(x)$ at $x = 1$.
- c) Find an equation of the tangent line of $f(x)$ at $x = 1$.

4. Let $f(x) = x^3 + 6x^2 - 15x - 20$. Using calculus (not the calculator)

- a) Find the intervals of increase and decrease of f .
- b) Find the relative (local) maxima and minima of f and their coordinates, if any.
- c) Find the intervals of upward and downward concavity of f .
- d) Find the inflection point(s) of f , if any.
- e) Graph $f(x)$ using the information from parts a-d and label all relative extrema and inflection points.

(continued on the back)

5. The demand for x number of units is given by the price function $p(x) = -\frac{1}{10}x^2 + 8x$ and the total cost function for x number of units is given by $C(x) = 2x^2 + 300$.
- Find the total revenue function, $R(x)$.
 - Find the total profit function, $P(x)$.
 - How many units need to be sold to maximize the profit? What is the maximum profit?
 - The point of diminishing returns (an inflection point) is the point at which the rate of growth of the profit function begins to decline. Find the point of diminishing returns.
6. Suppose that the profit function of a certain item, measured in dollars, is given by
- $$P(x) = -20.48x^{-2} - 0.27x + 300 \quad \text{for } 1 \leq x \leq 7$$
- where x is the quantity of the item.
- Use marginal analysis to estimate the profit for producing the 4th item.
 - Find the actual profit for producing the 4th item.
 - Find the absolute maximum profit and absolute minimum profit of the function on the specified interval.
 - Could we have used the Intermediate Value Theorem to predict we would have a profit of \$285 at some point in the interval? Explain.
7. Bank A pays 3.1% interest compounded monthly and Bank B pays 3% interest compounded continuously.
- If you invest \$1000 in each bank for 8 years, which account would be worth more? How much more?
 - If you invest \$1000 in Bank B, how long would it take you to reach \$1500 ?
8. An airplane flies horizontally in the sky at a constant height of 15 miles above the ground. It is on a flight path that will eventually take it directly over a child on the ground. If the diagonal distance between the airplane and the child is currently 39 miles and decreasing at a rate of 250 miles/hour, find the speed of the plane.