

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
 $2\frac{1}{2}$ HOURS

Mathematics 141

Spring 2026

Instructions: Answer all questions. Show all work.

1. Use algebraic methods to evaluate **all** of the following limits. (Note: If the limit is infinite, indicate whether it is $\pm\infty$)

(a) $\lim_{\theta \rightarrow 0} \frac{\theta + 1}{\cos \theta}$

(b) $\lim_{x \rightarrow \infty} \frac{(\sin \pi)x + (\cos 2\pi)x^2 + (\sin 3\pi)x^3 + (\cos 4\pi)x^4}{(\cos \pi)x + (\sin 2\pi)x^2 + (\cos 3\pi)x^3 + (\sin 4\pi)x^4}$

(c) $\lim_{x \rightarrow 5} \frac{3 - \sqrt{3x - 6}}{2x - 10}$

(d) $\lim_{x \rightarrow -1} \left[\frac{2}{x^2 - 1} + \frac{1}{x - 1} \right]$

2. Determine $\frac{dy}{dx}$ for **each** of the following. (Use any method.)

(a) $f(x) = (\sqrt{\pi + x})^\pi$

(b) $f(x) = \frac{\sin(\cos x) - x \tan x}{x}$

(c) $f(x) = (x\sqrt{x} \csc x)^{2/3}$

(d) $y^2 x^3 = \frac{x}{2y}$

3. Using only the definition of the derivative, find $f'(x)$ for $f(x)$ below.

$$f(x) = \frac{1}{x^2 + 1}$$

4. Let $f(x) = \begin{cases} cx + d & \text{if } x < -1 \\ d - x^3 + 2 & \text{if } -1 \leq x \leq 2 \\ c + 1 & \text{if } x > 2 \end{cases}$

Find the value of c and d such that $f(x)$ is continuous for all x .

5. Let $f(x) = \frac{x + \cos x}{1 - \sin x}$. Use the Intermediate Value Theorem to show that $f(x)$ has a root between $x = -\frac{\pi}{2}$ and $x = 0$.

(continued on the back)

6. Let $f(x) = \frac{x^2 + 4x + 5}{2c + 1} + \frac{c}{x}$.
- Find all values of c so that $f(x)$ has a horizontal tangent line at $x = 1$.
 - What is an equation of the tangent line to $f(x)$ (in part (a)) when $x = 1$?
7. Consider $f(x) = \frac{x^2 + 4x + 5}{2c + 1} + \frac{c}{x}$ with $c = 5$. If $f(x)$ models the displacement of an object at a time x , what is the velocity of the object at time $x = 3$?
8. For each of the following parts, refer to the function $f(x) = \frac{x^2}{1 - x^2}$. Using calculus,
- find all x -intercepts of $f(x)$.
 - find all y -intercepts of $f(x)$.
 - write an equation for any vertical asymptotes of $f(x)$, if they exist.
 - write an equation for any horizontal asymptotes of $f(x)$, if they exist.
 - find all intervals of increase and all intervals of decrease for $f(x)$.
 - find all intervals where $f(x)$ is concave up and all intervals where $f(x)$ is concave down.
 - Using your answers from parts (a) through (f), sketch the graph of $f(x)$.
9. A car is a vehicle with four wheels, while a motorcycle is a vehicle with two wheels. You own a store that sells cars and motorcycles, and the floorspace of your store can fit 100 wheels. The amount of money you make from selling these vehicles is given by the formula $Revenue = \frac{1}{3}C^3 - \frac{7}{2}C^2 + 10C - MC$, where C represents the number of cars in your store, and M represents the number of motorcycles in your store. Use calculus to determine what values of C and M will minimize the *revenue* that you make from selling these vehicles.
10. On a coordinate map, Charizard lives at point $(40, 40)$ and Bulbasaur lives at point $(0, 0)$. At the same time, Charizard begins walking west at a rate of 24 coordinate points per minute, and Bulbasaur starts walking east at a rate of 18 coordinate points per minute. How fast are the two moving apart when the Charizard reaches the y -axis at point $(0, 40)$?

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