

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
 $2\frac{1}{2}$ HOURS

Mathematics 142

Spring 2026

Instructions: Answer all questions. Show all work.

1. Find f if $f''(x) = \cos x - \sin x$, $f'\left(\frac{3\pi}{2}\right) = -1$, and $f\left(\frac{\pi}{2}\right) = 0$.
2. Differentiate the function.
 - a) $f(x) = \log_5(x^5 + 1)$
 - b) $f(t) = \sin^{-1}(5^t)$
 - c) $f(\theta) = e^{\tan^2 \theta} + e^{\tan^{-1} \theta}$
 - d) $y = x^{\sin x}$
 - e) $F(x) = \int_{\pi/2}^{\sqrt{x}} \sin(t^2) dt$
3. Integrate the function.
 - a) $\int \frac{x^2 - x + 1}{\sqrt[3]{x}} dx$
 - b) $\int \frac{12}{3 - x} dx$
 - c) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$
 - d) $\int \frac{dx}{16 + 25x^2}$
 - e) $\int_0^{\pi/2} \sin(3x) \cos^3(3x) dx$
 - f) $\int_1^e \frac{(\ln x)^2}{x} dx$
 - g) $\int_0^{\pi/3} \sqrt{4 \sec x + 1} \sec x \tan x dx$
4. A particle moves along a straight line so that its velocity at time t is $v(t) = t^2 - 3t + 2$ (measured in feet per second.)
 - a) Find the *displacement* of the particle during the time period $0 \leq t \leq 4$.
 - b) Find the *total distance* traveled by the particle during this period.
5. The half-life of radioactive Calcium-47 is 113 hours. Suppose we have a 100 mg sample. How much of the sample will remain after 62 hours? Round your answer to the nearest hundredth.
6. The region bounded by $x = (y - 1)^2$ and $x = y + 1$ is displayed on the right.
 - a) Find the area of the region.
 - b) Set up an integral using the method of cylindrical shells that gives the volume of the solid obtained by rotating the region about the x -axis. **(DO NOT EVALUATE THE INTEGRAL.)**
 - c) Set up an integral which gives the perimeter of the region. **(DO NOT EVALUATE THE INTEGRAL.)**
7. Find the solution of the differential equation $\frac{dy}{dx} = \frac{y \sec^2 x}{1 + 2y^2}$ that satisfies the initial condition $y(0) = 1$.
No need to solve for y .

