QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 143 Spring 2026

Instructions: Show your work. Phones should be turned off and away.

1. Compute:
   (a) $\displaystyle\int \frac{2x-2}{x^2-8x+20}\,dx$
   (b) $\displaystyle\int x^2\cos x\,dx$
   (c) $\displaystyle\int (\sin^3 4x)(\cos^2 4x)\,dx$
   (d) $\displaystyle\int \frac{dx}{x^2\sqrt{36-x^2}}$
   (e) $\displaystyle\int \frac{3x^2+x+8}{x(x^2+4)}\,dx$

2. Find the exact value of the following limit:
$$\lim_{x\to\infty}(1+\frac{1}{x})^{x/2}$$

3. Determine, without the use of a calculator, whether or not each of the following sequences converges or diverges. If a sequence converges, find what it converges to. If a sequence diverges, state that. Justify your answer in each case.
   (a) $\left\{\dfrac{(m^2)(m!)}{(m+2)!}\right\}$
   (b) $\left\{\dfrac{(-1)^n\cos(5n)}{n^2}\right\}$

4. Determine if each of the following series converges or diverges. Justify your answer in each case.
   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{4n}{n^2-3n+6}$
   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{5}{1+n2^n}$
   (c) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^5}$
   (d) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin n}{n^3}$
   (e) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2+n+1}{n^2-n-3}$
   (f) $\displaystyle\sum_{n=1}^{\infty}\frac{5^{3n+1}}{9^{2n}}$

(continued on the back)

5. Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x+4)^n}{\sqrt{n}}.$$

6. Determine if the following integrals converge or diverge.

   (a) $\displaystyle\int_0^2 \frac{1}{(x-2)}\,dx$

   (b) $\displaystyle\int_1^\infty \frac{1}{(1+x)^{3/2}}\,dx$

7. Using the Maclaurin series for $f(x) = \cos x$, compute the Maclaurin series for $g(x) = x^3 \cos(2x)$. Write your answer in summation notation.

8. (a) Compute $T_3(x)$, the third Taylor polynomial, for $f(x) = \ln x$ about $a = 1$.

   (b) If you use your answer in part (a) to estimate $f(x)$ on the interval [1,1.2], estimate the maximum error you can make using the formula for R_n.