

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
 $2\frac{1}{2}$ HOURS

Mathematics 151

Spring 2026

Directions: **SHOW ALL WORK** for each question in the blue booklet. Unless otherwise specified, leave answers in exact form.

1. Without using your calculator, evaluate each of the following limits. If the limit is ∞ , $-\infty$, or does not exist, explain why.

a. $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{2x^2 - 8}$

b. $\lim_{x \rightarrow 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x}$

c. $\lim_{x \rightarrow \infty} \frac{(2x+3)(2-5x)}{x^2 - x - 11}$

d. $\lim_{x \rightarrow -4^-} \frac{|x-2|}{x+4}$

e. $\lim_{x \rightarrow 0} \frac{\sin 3x - 4x}{5x}$

2. If $f(x) = \sqrt{1-8x}$

a. Using the definition of the derivative, find $f'(x)$

b. Write an equation of the line tangent to the graph of $f(x)$ at the point where $x = -1$.

3. In each of the following, find $\frac{dy}{dx}$. (Note: Algebraic simplification is not necessary.)

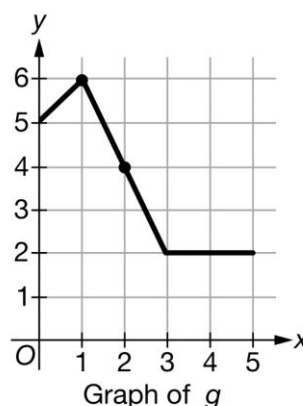
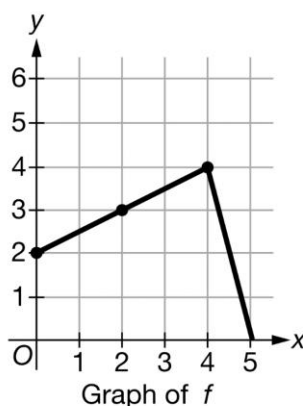
a. $y = \cos \sqrt{1-7x^4}$

b. $y = x^3 \sin^2(\pi x)$

c. $x^2 y^3 + 5y = 8x$

d. $y = \int_{\tan x}^{10} \sqrt[3]{6 + \sqrt{t}} \, dt$

4. The graphs of f and g are shown below. If $h(x) = \frac{f(x) + 1}{g(x) + 3x}$, find $h'(2)$.



(continued on the back)

5. Find the values of a and b such that $f(x)$ is differentiable for all x if

$$f(x) = \begin{cases} ax + b & x > -1 \\ bx^2 - 3 & x \leq -1 \end{cases}$$

6. We want to build a box (with a top) whose base length is 6 times the base width and the box will enclose 192 in^3 . The cost of the material for the sides is $\$3/\text{in}^2$ and the cost of the top and bottom is $\$7/\text{in}^2$. Determine the dimensions of the box that will minimize the cost. Include exact values and their units.
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7. Asha and Bryant are driving to Queens College. Asha is traveling east at a rate of 50 mph and Bryant is traveling south at 38 mph. At what rate is the distance between their cars changing when Asha is 1 mile and Bryant is $\frac{3}{4}$ mile from Queens College?
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8. If the velocity of a particle is given by $v(t) = 6t^2 - 2t - 3$ meters per second and the position of the particle at 2 seconds is 7 meters, what is the position of the particle at 4 seconds?
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9. Evaluate each indefinite integral

a. $\int \left(\frac{x^3}{3} + \frac{3}{x^3} - \sqrt[3]{x} + \sqrt{x^3} \right) dx$

b. $\int (2x - 1) \sec^2(5x^2 - 5x) dx$

c. $\int \frac{5x}{\sqrt{1 - 3x^2}} dx$

10. Below is the limit of a Riemann sum:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{2}{3} \left(\left(16 + \frac{6i}{n} \right) - 19 \right) \right) \left(\frac{6}{n} \right)$$

- a. Rewrite this limit as a definite integral.
b. Using analytic methods and the Fundamental Theorem of Calculus, find the value of the definite integral in part (a).