QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 152 **Spring 2026**

Instructions: Answer all questions. Show all work.

1. Evaluate each of the following limits using L'Hospital's Rule:

   (a) $\lim_{x \to 0} \dfrac{x^2}{1 - \cos x}$

   (b) $\lim_{x \to 0} \left(1 + \dfrac{4}{x}\right)^x$

2. Find the derivative of each of the following functions:

   (a) $f(x) = \ln(\tan^{-1} x) + \tan^{-1}(\ln x)$

   (b) $f(x) = e^{2x} \sin^{-1}(x^2)$

3. Evaluate each of the following integrals:

   (a) $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx$

   (b) $\displaystyle\int (6x + 2)e^{3x^2+2x}\,dx$

   (c) $\displaystyle\int \frac{5}{25 + x^2}\,dx$

   (d) $\displaystyle\int \sin^7 x \cos^5 x\,dx$

   (e) $\displaystyle\int \frac{1}{x^3\sqrt{x^2 - 1}}\,dx$

   (f) $\displaystyle\int \frac{5x + 3}{(x - 3)(x + 2)}\,dx$

4. Evaluate each of the following improper integrals:

   (a) $\displaystyle\int_1^{\infty} \frac{\ln x}{x^4}\,dx$

   (b) $\displaystyle\int_0^2 \frac{1}{2 - x}\,dx$

5. Let R be the region bounded by the curves $y = x^2$ and $y = 2 - x^2$.
   (a) Find the area of the region R.
   (b) Find the volume of the solid generated by rotating the region about the x-axis.

(continued on the back)

6. Determine whether each of the following series is conditionally convergent, absolutely convergent or divergent:

(a) $\sum_{n=1}^{\infty}(-1)^{n-1}n^{-4}$

(b) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(n+1)3^n}{2^{2n+1}}$

7. Find the radius and interval of convergence for the following series: $\sum_{n=1}^{\infty}\frac{(x+2)^n}{n4^n}$

8. Evaluate $\int x\tan^{-1}x\,dx$ by first rewriting the function as an infinite series.

9. Use Simpson's rule with $n=6$ to approximate $\int_2^4 e^{-x^2}dx$

10. Solve the differential equation: $\frac{dy}{dx}+2xy=y$

11. Find the sum of the series: $\sum_{n=1}^{\infty}\frac{2^{2n+1}}{7^n}$