QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 115   FALL 2019

Instructions: For every solution, please *show all of the work* in the blue book provided.

1. Given two points $P(-3, 1)$ and $Q(2, -3)$:

    (a) Find the midpoint of line segment $\overline{PQ}$.
    (b) Find the length of line segment $\overline{PQ}$.
    (c) Write an equation of a line perpendicular to $\overline{PQ}$ passing through its midpoint.
    (d) Write an equation of a circle whose center is the midpoint of $\overline{PQ}$ and whose diameter is $\overline{PQ}$.

2. Factor the following completely:

    (a) $12a^4b - 27a^2b^3$
    (b) $10x^2 - 25x - 60$
    (c) $5pq + 15pr - q - 3r$

3. Solve the following system of linear equations:

$$\begin{cases} 2x - 5y & = -16 \\ 3x + 2y & = -5 \end{cases}$$

4. Simplify the following expressions:

    (a) $\dfrac{1 - \dfrac{3}{x}}{1 - \dfrac{2}{x} - \dfrac{3}{x^2}}$

    (b) $(\sqrt{7x} - 3)^2 - (\sqrt{7x-3})^2$

    (c) $x^2y\sqrt{20x^5y^7} - 2xy\sqrt{5x^7y^7} + 7xy^2\sqrt{45x^7y^5}$

    (d) $\dfrac{(2a^{-5}b^{3/2})^{-4}(8a^6b^{-9})^2}{(a^4b^{-8})^{-1/2}}$ (answer should involve positive exponents only)

5. Find all <u>real</u> solutions of x for the following equations:

    (a) $x(x+2) = 3$
    (b) $4 + \sqrt{5 - 2x} = x + 3$
    (c) $\dfrac{x-7}{x^2 - 4x - 5} + \dfrac{3x - 13}{x - 5} = \dfrac{2x}{x+1}$
    (d) $x^2 - 8x + 9 = 0$

6. Perform polynomial long division to find the quotient and remainder: $(2x^3 - 13x - 10) \div (x - 3)$

7. Rationalize and simplify: $\dfrac{8}{\sqrt{5} - 3} + \dfrac{10}{\sqrt{5}}$

8. Sketch the graph of $7x - 3y = 21$ and label its intercepts on the graph.

9. Divide: $\dfrac{2x^2 - 3x - 9}{4x^2 - 12x} \div \dfrac{3x^2 - 6x}{x^3 - 2x^2}$

10. For the function $f(x) = x^2 - 8x - 9$, find the following:

    (a) an equation of the axis of symmetry

    (b) coordinates of the vertex

    (c) $x-$intercept(s) & $y-$intercept

11. If $f(x) = 3 - 7x$ and $g(x) = 5x^2 - 3x + 4$, evaluate:

    (a) $[f(-1)]^2$

    (b) $g(3a)$

    (c) $f(a + 2)$

12. Find the domain of each of the following functions. Express your answer using interval notation.
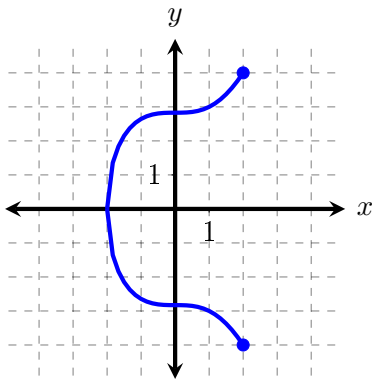
    (a) $f(x) = 3x^2 + 7x - 5$

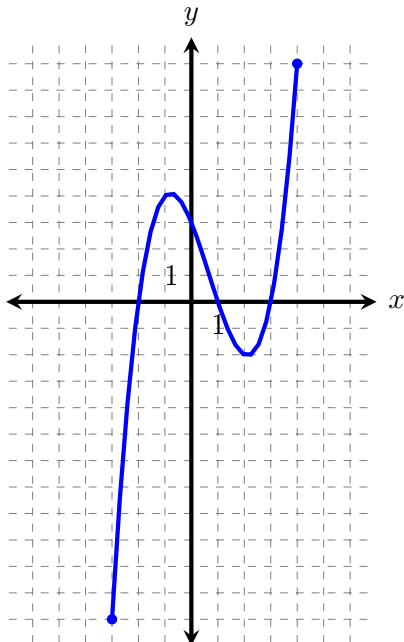   (b) $g(x) = \sqrt{3 - 8x}$

    (c) $h(x) = \dfrac{1}{\sqrt{x - 2}}$

13. Determine whether or not the relation represents a function <u>and</u> explain your reasoning. Write the domain and range of each relation.

    (a) $\{(-10, 4), (3, 7), (10, 3), (9, 7), (1, 1)\}$

    (b) The following graph:

14. The graph of $y = f(x)$ is shown below.

Use the graph of $y = f(x)$ to find the following:

    (a) domain of $f(x)$ (in interval notation)

    (b) range of $f(x)$ (in interval notation)

    (c) $f(0)$

    (d) the values of x where $f(x) = 0$

15. An amusement park charges $35 for adults and $14 for children. A party of eleven paid $175, in total, for admission into the park. How many adults and how many children were there in this group?