QUEENS COLLEGE
MATHEMATICS DEPARTMENT

FINAL EXAM
$2\frac{1}{2}$ HOURS

Mathematics 122 **Fall 2015**

INSTRUCTIONS: ANSWER ALL QUESTIONS. <u>SHOW ALL WORK</u>

1) a) Given two points $P(5,1)$ and $Q(3,-3)$:
   i) Find an equation for line $\overleftrightarrow{PQ}$.
   ii) Find an equation for the line perpendicular to $\overleftrightarrow{PQ}$ that passes through the point $(2,4)$.
   iii) Find an equation for the circle that has points P and Q as endpoints of a diameter.

b) Show that the equation $x^2 + 6x + y^2 - 4y - 3 = 0$ represents a circle and find the center and radius of the circle. Then sketch its graph.

2) a) Given the function $f(x) = x^2 + 3x$, compute i) $f(-2)$ ii) $f(1)$

b) Given the piecewise function $g(x) = \begin{cases} 2 & \text{if } x \le -1 \\ 3x - 1 & \text{if } x > 1 \end{cases}$, compute
   i) $g(-3)$ ii) $g(-1)$ iii) $g(2)$

c) For the function $F(x) = x^2 + 4x + 3$, find
   i) $F(a)$
   ii) $F(a+h)$
   iii) the difference quotient $\frac{F(a+h)-F(a)}{h}$, $h \neq 0$

3) Sketch a graph of each function by starting with the graph of a standard function and applying transformations. Label the coordinates of any vertex, x-intercept(s), and y-intercept. Write an equation of any vertical and horizontal asymptote where appropriate.

a) $f(x) = x^2 - 4$
b) $f(x) = \sqrt{x-3}$
c) $f(x) = -|\,x+1\,| - 2$
d) $f(x) = \frac{1}{x-2} + 1$

4) a) For $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{2}{x}$, find the composite functions $(f \circ g)(x)$ and $(g \circ f)(x)$. For each composite, find its domain.

b) For $H(x) = \frac{x^2}{x^2+3}$, find functions $f(x)$ and $g(x)$ such that $(f \circ g)(x) = H(x)$.

c) For $F(x) = \frac{2}{x-1}$,
   i) find $F^{-1}(x)$
   ii) use the inverse function property to show that $(F \circ F^{-1})(x) = (F^{-1} \circ F)(x)$

5) Given the quadratic function $f(x) = x^2 + 4x + 5$:
a) Write $f(x)$ in standard form and find its vertex.
b) Sketch the graph of $f(x)$.
c) Find the minimum value of $f(x)$.

(continued on the back)

6) **A rectangular field is to be constructed using 600 feet of fencing. An interior fence that runs parallel to one of the exterior fences divides the field into two parts.**

a) Find a function that models the area of the field in terms of its width x.

b) What are the dimensions of the field that has the largest area?

c) What is the maximum area of the field?

7) **Sketch a graph of the polynomial function $f(x) = x^3 - x^2 - 12x$. Make sure the graph shows all intercepts. Indicate its behavior as x gets very positive and as x gets very negative.**

8) **a) Without the use of a calculator, evaluate $\log_6 12 + \log_6 8 - 2\log_6 4$**

b) Write $\log \frac{x^2\sqrt{3x-1}}{x+2}$ as a sum & difference of logarithms.

c) Solve for x: $\log_2(x-3) + \log_2(x-6) = 2$

d) Solve for x: $81^{1-x} = 27^{x-1}$

e) Sketch the graph of $f(x) = 2^{x+1} - 1$. Label its x- and y-intercept and its asymptote.

9) **a) Without the use of a calculator, evaluate $\sin 43° \cos 17° + \cos 43° \sin 17°$**

b) Prove the following trigonometric identities:

i) $\tan x + \cot x = \sec x \csc x$

ii) $\frac{\sec^2 x - 1}{\tan x \sin x} = \sec x$

c) Sketch the graph of $f(x) = 3\cos 2x$ on the interval $[0,\ 2\pi]$. Label all x- and y-intercepts of the graph.

10) **Given $\cos A = -\frac{8}{17}$, where $\measuredangle A$ is in quadrant II, and $\tan B = \frac{3}{4}$, where $\measuredangle B$ is in quadrant III, find:**

a) $\tan A$

b) $\cos(A+B)$

c) $\sin 2B$