

**QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS**

Final Examination

$2\frac{1}{2}$ Hours

Mathematics 122

Fall 2017

Instructions: Answer all the questions and show all work in the blue book.

1. Given $f(x) = \sqrt{x+9}$ and $g(x) = (x-1)^2$.
 - a) Sketch the graph of $f(x)$.
 - b) Use the graph of $f(x)$ to sketch the graph of $f^{-1}(x)$ on the same coordinate axes. Show clearly x - and y -intercepts.
 - c) Find an equation for $f^{-1}(x)$ and determine the domain and range of $f^{-1}(x)$.
 - d) Evaluate $(g \circ f)(27)$.

2. Let $f(x) = 3 - 4x^2$ and $g(x) = 5x - 6$.
 - a) Find $\frac{f(a+h)-f(a)}{h}$ for $h \neq 0$, and simplify.
 - b) Find $(f \circ g)(x)$ and simplify.

3. Determine the domain for each of the following functions.
 - a) $h(t) = \frac{t+7}{-t^2+t+2}$
 - b) $y = \frac{4-x}{\sqrt{4x+2}}$
 - c) $g(x) = 3 \log(x-4)$
 - d) $h(\theta) = 2 \sin\left(\theta + \frac{\pi}{2}\right)$

4. Let $f(x) = -x^2 + 4x + 5$.
 - a) Express in standard form and find the vertex.
 - b) Sketch a graph of the function.
 - c) State clearly the maximum or minimum value, x - and y -intercepts, and the domain and range of the function.

5. Evaluate without using a calculator and show all work.
 - a) $\sin^{-1}\left(\tan \frac{\pi}{4}\right)$
 - b) $\sec\left(\frac{16\pi}{3}\right)$
 - c) $\log 2 - \log 5 + \log 250$
 - d) $\cos 100^\circ \cos 70^\circ + \sin 70^\circ \sin 100^\circ$
 - e) $e^{2 \ln 5}$

(continued on the back)

6. Solve for x (Round answers to 5 decimal places where necessary.)
- $9 = e^{2x}$
 - $\log_2(x - 1) + \log_2(x + 2) = 2$
 - $5^x = 3^{x+1}$
 - $2 \cos^2 x = \sin x + 1, [0, 2\pi]$
7. Sketch the graph of each function. Show clearly all intercepts and asymptotes if applicable.
- $y = e^{x+1} + 2$
 - $g(\theta) = 3 \cos(2\theta)$ on the interval $[0, 2\pi]$
 - $y = 8 - (x - 2)^3$
 - $y = \frac{-1}{x-2} + 1$
8. Verify each of the following trigonometric identities:
- $2 \tan x \sec x = \frac{1}{1-\sin x} - \frac{1}{1+\sin x}$
 - $\csc x \sec x = \tan x + \cot x$
9. If $\cos A = -\frac{4}{5}$ where A is in Quadrant III, and $\sin B = -\frac{5}{13}$ where B is in Quadrant IV. Evaluate the following:
- $\cos 2A$
 - $\sin(A - B)$
 - $\sec B$
 - $\tan 2B$
10. A farmer has 2,400 ft of fencing and wants to fence off a rectangular area and divide it into four pens with fencing parallel to one side of the rectangle (see the figure below.) Find a function that models the total area of the four pens in terms of x .

