QUEENS COLLEGE

DEPARTMENT OF MATHEMATICS

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 122 **Fall 2018**

Instructions: Answer all the questions and show all work in the blue book.

1. Let $f(x) = x^2$ and $g(x) = \sqrt{4 - x}$.
   a) Find $(g \circ f)(x)$ and its domain.
   b) Find $\dfrac{f(x+h) - f(x)}{h}$, where $h \neq 0$. Simplify your answer.

2. Given the function $g(x) = -2(x-5)^3 + 2$,
   a) identify the parent function f
   b) describe the sequence of transformations from f to g
   c) sketch the graph of g.

3. Solve the following equations algebraically:
   a) $e^{\ln x} = 3$
   b) $\log_3(2x - 5) + \log_3 x = 1$
   c) $4^{x-5} + 14 = 30$

4. Given the polynomial function $P(x) = 4x^3 + 4x^2 + x$,
   a) find all the zeros of P and determine the multiplicity of each zero
   b) sketch the graph of $y = P(x)$.

5. Find the exact value of each expression without using a calculator:
   a) $\tan(\cos^{-1}(\dfrac{\sqrt{2}}{2}))$
   b) $\sin(\dfrac{\pi}{6} + \dfrac{\pi}{4})$
   c) $\cos\dfrac{\pi}{4}\cos\dfrac{\pi}{12} + \sin\dfrac{\pi}{4}\sin\dfrac{\pi}{12}$

6. Verify the following trigonometric identity:
   $\dfrac{\csc x + \sec x}{\sin x + \cos x} = \cot x + \tan x$

7. In ΔABC, find the indicated quantity, rounding your answers to two decimal places:
   a) If $A = 36°$, $B = 98°$, $c = 16$, find b.
   b) If $A = 35°$, $b = 8$, $c = 12$, find a.

(continued on the back)

8. Sketch the graph of each of the following functions. Label the coordinates of all vertices, and intercepts, and equations of the asymptotes, if applicable.

   a) $f(x) = \ln(x - 2)$

   b) $g(x) = \cos(2x) + 1$ on the interval $[0, 2\pi]$

   c) $r(x) = \dfrac{4x - 4}{x + 2}$

   d) $f(x) = e^{x+5} - 1$

   e) $h(x) = -\sqrt{x + 4}$

9. The total revenue R (in dollars) earned from manufacturing and selling hand-held video games is given by $R(p) = -25p^2 + 1200p$, where p is the price per unit (in dollars).

   a) Find the unit price that will yield the maximum revenue.

   b) What is the maximum revenue?

   c) Use the results from part (a) and part (b) to sketch the graph of $R(p)$ indicating the coordinates of the vertex and all intercepts.

10. Solve the following trigonometric equation, where x lies in the interval $[0, 2\pi)$:

    $2\sin^2 x + 3\cos x - 3 = 0$