### Queens College Department of Mathematics

# Final Examination

### $2\frac{1}{2}$ Hours

Math 122

#### **INSTRUCTIONS:**

#### **ANSWER ALL QUESTIONS**

## SHOW ALL WORK

Fall 2019

1) Let  $f(x) = \frac{x-2}{x+1}$  and  $g(x) = \sqrt{3x+6}$ .

- **a)** Find the domain of *f* and the domain of *g*.
- **b)** Sketch the graph of y = f(x). Label all intercepts and asymptotes and determine the range of f.
- c) Find  $f^{-1}(x)$  algebraically and find its domain.
- **d)** Sketch the graph of y = g(x) and use it to sketch the graph of  $y = g^{-1}(x)$ , the inverse function, on the same set of axes. Label all intercepts.
- e) Find  $(f \circ g)(x)$  and find its domain.
- **2)** Let  $p(x) = 2x^2 8x + 9$ .
  - **a)** Express p(x) in standard form (vertex form).
  - **b)** Sketch the graph of y = p(x). Label the vertex, and any and all intercept(s).
  - c) State the minimum value of p(x).

#### **3)** Solve for x:

- **a)**  $64^{3x+1} = 8^{4x-2}$
- **b)**  $\tan(\cos^{-1}(\frac{\sqrt{2}}{2})) = 3x + 2$
- **c)**  $\log_2(6x+2) \log_2(x-3) = 4$

d)



(Leave your answer in simplest radical form)

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- 4) Let  $P(x) = x^3 3x^2 10x$ .
  - a) Factor the polynomial and use the factored form to find all of the zeros of *P*.
  - **b**) Describe the end behavior of this polynomial function.
  - **c)** Use your calculator to find the coordinates of all of the local extrema of *P*. (Round your answers to two decimal places.)
  - d) Sketch the graph of this polynomial function. Label any and all intercepts.
- 5) Find the exact value of each of the following without using a calculator:
  - **a)**  $\sin(70^\circ)\cos(10^\circ) \cos(70^\circ)\sin(10^\circ)$
  - **b)**  $\log_7(28) + \log_7(7) 2\log_7(2)$
  - **c)**  $\cos^2(\frac{\pi}{12}) \sin^2(\frac{\pi}{12})$
- **6)** Prove the following identity:

$$2\sec x \sin x - \frac{2\tan x}{\csc^2 x} = \sin 2x$$

- 7) Solve the equation  $2\sin^2(\theta) \cos(\theta) = 1$  in the interval [0,2 $\pi$ ).
- 8) If  $cos(A) = -\frac{12}{13}$ , where A is in Quadrant II, and  $tan(B) = \frac{8}{15}$ , where B is in Quadrant III, evaluate the following:
  - **a)**  $-\sin(2B)$
  - **b)**  $2\sin(A)$
  - c)  $\cos(A+B)$
- **9)** Sketch the graph of each of the following functions. In each case, determine its domain, label any and all intercepts, and identify any and all horizontal or vertical asymptotes.
  - **a)**  $f(x) = -e^{x+1} + 1$
  - **b)**  $g(x) = \frac{1}{3}\sin(x \frac{\pi}{4})$  (Show one complete period.)
  - c)  $h(x) = \ln(x+4) 2$
- **10)** Virginia invested \$17,000 in an account that pays 8% interest per year, compounded quarterly.
  - **a)** Find the amount in the account after 3 years. (Round your answer to two decimal places.)
  - **b)** How long will it take for the investment to double? (Round your answer to two decimal places.)