

QUEENS COLLEGE MATHEMATICS DEPARTMENT
FINAL EXAMINATION
2.5 HOURS

Mathematics 122
Answer all questions

Spring 2015
Show all work

1. Determine the domain for each of the following functions:

a) $f(x) = \sqrt{21 - 3x}$

d) $y = 3 \cos \left(5x + \frac{\pi}{2} \right)$

b) $y = \log (2x - 8)$

c) $h(x) = \frac{4x^2 - 8}{\sqrt{5x + 7}}$

e) $g(x) = \frac{8x + 12}{3x^2 - 13x + 10}$

2. Use algebra to determine the x- and y-intercepts and the vertical and horizontal asymptotes of the following rational function. Then sketch its graph.

$$R(x) = \frac{2x^2 + 4x}{x^2 - x - 2}$$

3. Given the quadratic function $F(x) = x^2 - 8x + 14$,

- complete the square to write the function in vertex form.
- identify the vertex and write an equation of its axis of symmetry.
- determine the zeros of the function and express them in simplest radical form.

4. Let $\sin A = \frac{4}{5}$, where angle A is in quadrant II, and let $\cos B = \frac{-\sqrt{5}}{7}$, where angle B is in quadrant III.

Determine the exact value of each of the following expressions. Simplify any radicals if necessary.

- $\sec A$
- $\cot^2 B + \tan^2 A$
- $\cos (2A)$
- $\sin (A + B)$
- $\cos (A + B)$

5. Given the polynomial function $p(x) = x^4 + x^3 - 3x^2 - x + 2$,

- use the Rational Zeros Theorem to list all possible rational zeros of $p(x)$.
- determine the zeros of $p(x)$ using any method. Then write $p(x)$ in factored form.
- sketch the graph of $p(x)$ and identify all intercepts.
- explain why $(x + 6)$ is not a factor of $p(x)$.

6. Given the functions $f(x) = x^2 - 4x$, $g(x) = \frac{2}{\sqrt{1+x}}$, and $h(x) = \frac{3}{x+5}$, find:

- $(h \circ g)\left(\frac{-16}{25}\right)$
- $f(g(99))$
- $(f \circ f)(x)$
- $h^{-1}(x)$ and verify your result using composition of functions.

(continued on the back)

7. Follow the directions for each part below.

- a) Solve for x : $2 \log_4(x) - \log_4(x + 3) = 1$
- b) Solve for x : $\frac{1}{125^{-2x}} = 25^{x+1}$
- c) Expand the logarithmic expression: $\log \frac{4x^5 \sqrt{8+7x}}{x-6}$
- d) Combine into a single logarithm: $\frac{1}{3} [\ln(x - 5) + \ln(2x)]$
- e) Evaluate without the use of a calculator: $2 \left(\ln e^\pi - \frac{\pi}{2} \right)$

8. a) Sketch the graph of the trigonometric function $y = 3 \sin 2x$ on the interval $-2\pi \leq x \leq 2\pi$.

Label the x -intercepts of the graph.

b) Given the trigonometric function $f(x) = -2 \cos \frac{\pi}{4} x$, find its amplitude, frequency, and period.

c) Given the trigonometric function $y = -\tan x$, write equations for any two of its asymptotes.

(Hint: Where is the function undefined?)

9. Evaluate each of the following expressions without the use of a calculator :

- a) $\sin \frac{-5\pi}{4}$
- b) $\cos \frac{10\pi}{3}$
- c) $\tan[\sin^{-1} \left(\frac{-\sqrt{3}}{2} \right)]$
- d) $\cos 97^\circ \cos 83^\circ - \sin 97^\circ \sin 83^\circ$

10. Use transformations to provide a reasonably accurate sketch of the graph of each of the functions below. Label all vertices, intercepts, and asymptotes.

For each function, indicate the appropriate "parent function" by writing its equation next to the corresponding graph.

- a) $f(x) = e^{-x} + 5$
- b) $y = (x + 2)^3 - 1$
- c) $g(x) = -|x - 5| + 4$
- d) $h(x) = \sqrt{x + 16} + 1$
- e) $s(x) = \frac{1}{x-2} + 3$
- f) $y = \log_2(x - 3) + 2$