Final Examination
2.5 Hours

Mathematics 122   **Spring 2016**

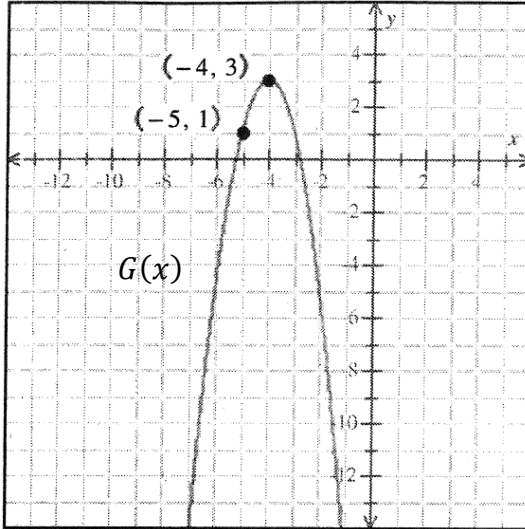**Instructions:** <u>Answer all the questions.</u> <u>Show all work.</u>

1. Let $A(3,1)$ and $B(-1,3)$ be two points in a plane
    a) Which of these points is closer to the point $C(-1,-1)$?
    b) Find an equation of a circle whose diameter has the endpoints A and B .
    c) Find an equation of the line that is parallel to the line joining points A and B and passes through the origin.

2. a) Find the composite function $(f \circ g)(x)$, given $f(x) = \frac{1}{x+1}$ and $g(x) = \frac{1}{3x-1}$.
    b) Given the function $f(x) = x^2$ where $x \geq 0$. Find $f^{-1}(2)$.
    c) Find $\frac{f(x+h)-f(x)}{h}$ if $f(x) = x^2 + 2x - 1$, where $h \neq 0$.

3. Sketch the graph of each of the following equations. Where appropriate, indicate center, radius, x- and y-intercepts, and asymptotes.
    a) $g(x) = -\log_2 x - 3$
    b) $h(x) = -3(x+1)^2(x-4)$
    c) $y = -2\sin\frac{1}{2}x$, where $0 \leq x \leq 4\pi$
    d) $f(x) = -(x-2)^2 - 4$
    e) $2x^2 + 2y^2 + 12x - 16y - 22 = 0$

4. Let $f(x) = \sqrt{x-3}$.
    a) Sketch the graph of $y = f(x)$.
    b) Find an equation for $f^{-1}(x)$.

5. Find an equation of the quadratic function G whose graph is shown at the right.

6. Evaluate without using a calculator:
    a) $\cos(\pi)\cos\left(\frac{3\pi}{4}\right) + \sin(\pi)\sin\left(\frac{3\pi}{4}\right)$
    b) $\cos^2(15°) - \sin^2(15°)$
    c) $\sin\left(\frac{10\pi}{3}\right)$

(continued on the back)

7.　　a)　Use the laws of logarithms to expand the expression $\log \sqrt[4]{x^2 + y^2}$

　　　b)　Find the domain of $g(x) = \log_{10}(x - 4)$ and sketch the graph of $y = g(x)$.

8.　　Solve for x:

　　　a)　$\log_2(x + 3) + \log_2(x + 5) = 3$

　　　b)　$3^{x+1} = 9^x$

　　　c)　$|5x - 8| = 32$

9.　　Verify each of the following trigonometric identities:

　　　a)　$\dfrac{\cos x}{1 - \sin x} - \dfrac{1}{\cos x} = \tan x$

　　　b)　$\sin x \sin(x + y) + \cos x \cos(x + y) = \cos y$

10.　　If $\tan A = -\dfrac{4}{5}$, where A is in Quadrant IV, and $\cos B = -\dfrac{5}{13}$, where B is in Quadrant II, evaluate the following:

　　　a)　$\cos 2A$

　　　b)　$\sin(A - B)$

　　　c)　$\csc^2 A$

　　　d)　$\sin A + \sec B$

11.　　An rectangular box with no top and with a square bottom has a volume of 150 cubic meters. Its bottom and sides are made from two different materials. It costs 8 dollars per square meter for the bottom material, and 10 dollars per square meter for the material for the sides. Express the total cost of building the box in terms of x, where x represents one of the lengths of the bottom side.