

**QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS**

Final Examination

$2\frac{1}{2}$ Hours

Mathematics 122

Spring 2018

Instructions: Answer all the questions and show all work in the blue book.

1. Let $f(x) = \frac{3}{x}$ and $g(x) = \sqrt{x-4}$.
 - a) Find $\frac{f(x+h)-f(x)}{h}$, where $h \neq 0$, and simplify.
 - b) Find $(f \circ f)(x)$ and its domain.
 - c) Find $g\left(f\left(\frac{1}{12}\right)\right)$. Express the answer in simplest radical form.

2. Given the function $f(x) = (x-1)^2, x \geq 1$.
 - a) Sketch the graph of $f(x)$ and determine its domain and range. Label all intercepts.
 - b) Use the graph of $f(x)$ to sketch the graph of $f^{-1}(x)$ on the same coordinate axes. Label all intercepts.
 - c) Express $f^{-1}(x)$ as a function of x and determine its domain and range.

3. A business has determined that the revenue (in thousands of dollars) generated by selling x units of merchandise (in thousands) is given by the function $R(x) = -\frac{1}{2}x^2 + 2x$.
 - a) Express the function in standard form.
 - b) Sketch the revenue function. Label the vertex, x - and y - intercept(s).
 - c) What is the maximum revenue?
 - d) How many units must be sold to obtain the maximum revenue?

4. Solve for x :
 - a) $3e^{7x} = 14$
 - b) $\log_3(x-8) + \log_3 x = 2$
 - c) $9^{\left(\frac{2}{\log_2(x)}\right)} = 3$

5. Given the polynomial function $P(x) = x^3 - 3x^2$.
 - a) Factor the polynomial and use the factored form to find all the zeros.
 - b) Describe the end behavior of the polynomial function.
 - c) Use a calculator to find the coordinates of all the local extrema.
 - d) Sketch the graph of the polynomial function. Label x - and y -intercept(s).

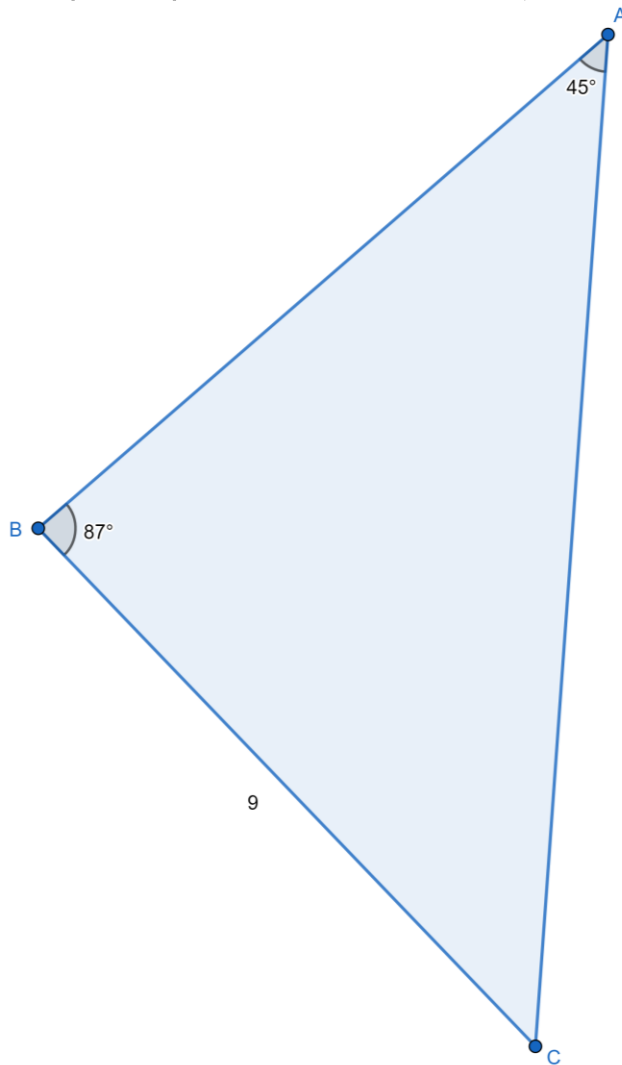
6. Evaluate each of the following expressions without the use of a calculator:
 - a) $\tan\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$
 - b) $\sec\left(\frac{5\pi}{6}\right)$
 - c) $\cos\frac{\pi}{6}\cos\frac{\pi}{12} - \sin\frac{\pi}{6}\sin\frac{\pi}{12}$
 - d) $\log_5 5 + \log_5 20 - \log_5 4$

(continued on the back)

7. Sketch the graph of each of the following functions. Label the coordinates of all vertices, intercepts, and the equations of the asymptotes, if applicable.
- a) $f(x) = 2 - \sqrt{x+4}$
 - b) $g(x) = \frac{1}{3} \sin(2x)$ on the interval $[0, 2\pi]$
 - c) $h(x) = 1 - \ln(x)$
 - d) $k(x) = \frac{1}{x+3} - 1$

8. Verify the following trigonometric identity: $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 2 \sec x$.

9. Solve the following triangle using the Law of Sines. (Round your answers to one decimal place.)



10. Solve the equation $2 \cos^2 \theta - 1 = 0$, where θ is in the interval $[0, 2\pi)$.