QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 122 **Spring 2019**

Instructions: Answer all the questions. Show all work.

1. Let $f(x) = \frac{10}{x-7}$ and $g(x) = \sqrt{x+4}$.
   a) Find the domain of f and the domain of g.
   b) Sketch the graph of $y = g(x)$ by performing a transformation of $y = \sqrt{x}$. Label the x and y intercepts and determine the range of g.
   c) Use the graph of $y = g(x)$ to sketch the graph of $y = g^{-1}(x)$. Label the graph and determine the x and y intercepts of $g^{-1}(x)$.
   d) Find $g^{-1}(x)$ algebraically and find its domain and range.
   e) Find $(f \circ g)(x)$.

2. Let $f(x) = 3x^2 - 12x - 2$.
   a) Express $f(x)$ in standard form.
   b) Sketch the graph of $y = f(x)$. Label the vertex, and the y intercept.
   c) State the maximum or minimum value of $f(x)$.

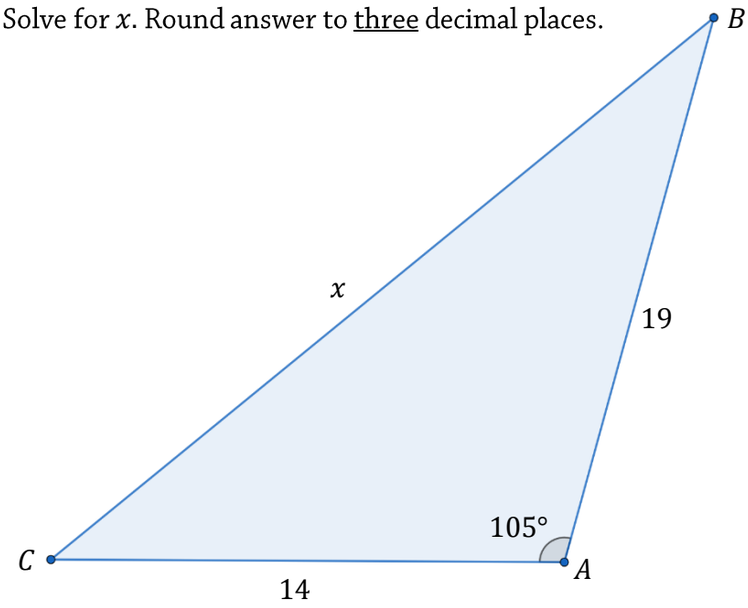
3. a) Find the inverse function of $f(x) = 7 - 5x$ and verify that $f^{-1}(f(x)) = x$.
   b) Given $g(x) = 5x^2 - 7x + 11$, find $\frac{g(a+h) - g(a)}{h}$, where $h \neq 0$, and simplify.

4. Sketch the graphs of the following equations. Label the coordinate of all vertices, intercepts, and the equation of the asymptotes, if applicable.
   a) $y = -(x+2)^3$
   b) $y = |x-7| - 10$
   c) $y = \log_6(x-3)$
   d) $y = 5\sin 2x$ on the interval $[0, 2\pi]$
   e) $y = \frac{-4}{x+2} - 1$

5. <u>Without using your calculator</u>, solve for x:
   a) $4^{x+8} = 32^{x-1}$
   b) $\log_2(x+5) + \log_2(x-1) = 4$

(continued on the back)

6. Solve for x. Round answer to <u>three</u> decimal places.



7. Let A be a second quadrant angle with $\sin A = \frac{12}{13}$ and let B a third quadrant angle with $\cos B = -\frac{4}{5}$. Find:
   a) $\cos(A - B)$
   b) $\sin(2B)$
   c) $\sec^2(A)$

8. <u>Without using your calculator</u>, find the exact value of each of the following expressions:
   a) $\sin 27° \cos 33° + \cos 27° \sin 33°$
   b) $\tan\left(\frac{25\pi}{6}\right)$
   c) $\csc\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$
   d) $\log_3 27 + \log_3 5 - \log_3 15$

9. Verify the following identity: $\frac{1 - \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} = 2 \csc x$

10. A man invests \$3,000 in an account that pays 7.5% interest per year, compounded semi-annually.
    a) Find the amount after 4 years. Round answer to <u>two</u> decimal places.
    b) How long will it take for the investment to grow to \$7,000? Round answer to <u>one</u> decimal place.