QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

Mathematics 122 **Spring 2022**

Instructions: Answer all questions. Show all work.

1. Consider the piecewise function

$$f(x) = \begin{cases} 5 & \text{if } x < -3 \\ x^2 & \text{if } -3 \le x < 2 \\ x+1 & \text{if } x \ge 2 \end{cases}$$

a) Evaluate $f(6)$, $f(-6)$, $f(-1)$, and $f(-3)$.
b) Sketch the graph of $y = f(x)$.

2. Find the domain of $f(x) = \frac{x-1}{2x^2+9x-5}$. Express your answer in interval notation.

3. Given $f(x) = x^2 - 3x + 2$, find and simplify:
a) $f(a)$
b) $f(a+h)$
c) The difference quotient, $\frac{f(a+h)-f(a)}{h}$, where $h \neq 0$.

4. a) Given the graph of $y = f(x)$, explain what transformations you would do to obtain the graph of
(i) $y = f(x-2)$.
(ii) $y = -f(x) - 2$.

b) Let $f(x) = \sqrt{x+9}$.
(i) Find the domain of $f(x)$.
(ii) Using appropriate transformations to sketch the graph of $f(x)$. Label the coordinates of all intercepts of the graph.

c) Starting with the graph of $y = g(x) = \frac{1}{x}$, use appropriate transformations to sketch the graph of $y = h(x) = \frac{1}{x+2} + 1$. Label the coordinates of any and all intercepts of the graph of h and write an equation of any horizontal and vertical asymptotes of the graph of h.

5. Let $f(x) = \frac{x}{x-1}$ and $g(x) = \frac{1}{x}$. Find $f \circ g$ and simplify.

6. Consider the function $f(x) = \frac{3}{x-2}$.
a) Find the domain of $f(x)$.
b) Find f^{-1}, the inverse function of f, and its domain.
c) Using the result of part b), show that $f^{-1}\big(f(1)\big) = 1$ and $f\big(f^{-1}(1)\big) = 1$.

(continued on the back)

7. A quadratic function f is given to be $f(x) = x^2 - 4x - 5$. (Only algebraic solutions will be accepted.)
   a) Express f in vertex form.
   b) Find the coordinates of the vertex.
   c) Find the x- and y-intercepts of f.
   d) Sketch a graph of f.
   e) Find the minimum or maximum value of f. Is this a minimum or maximum value?

8. Solve each of the following equations for x:
   a) $\left(\frac{1}{32}\right)^{x-1} = 4^{2-3x}$

   b) $\log_3 x + \log_3(x-2) = 1$

   c) $7^{1-x} = 3$ Round answer to three decimal places.

9. Using the laws of logarithms, not a calculator, evaluate the expression $\log_3 6 - \log_3 10 + \log_3 45$.

10. a) WITHOUT USING YOUR CALCULATOR, find the exact value of $\sin\frac{7\pi}{12}$ by using an appropriate addition or subtraction formula.

    b) WITHOUT USING YOUR CALCULATOR, find the exact value of $\cos\frac{\pi}{4}\cos\frac{\pi}{12} + \sin\frac{\pi}{4}\sin\frac{\pi}{12}$ by using an appropriate addition or subtraction formula.

11. If $\tan\theta = -\frac{2}{3}$ and $\sin\theta < 0$, find
   a) $\sin 2\theta$
   b) $\cos 2\theta$

12. Verify the identity $\cos\theta\cot\theta + \sin\theta = \csc\theta$.

13. Find the amplitude and period of the function $f(x) = 4\sin\left(\frac{1}{3}x\right)$ and graph one complete cycle of it.

14. Solve for θ, where θ is in the interval $[0,2\pi]$:
$$\sin\theta\,(2\sin\theta + 5) = 3$$