

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
FALL 2015

Math 131

2 1/2 Hours

INSTRUCTIONS: ANSWER ALL QUESTIONS. SHOW ALL WORK

1. The profit made in manufacturing and selling x thousand television sets has been determined to be $P(x) = 9x^3 - x^4 + 5x - 200$ hundred dollars. Due to production limitations, a maximum of 8000 television sets can be produced.

- a) Sketch the graph of $P(x)$ for $0 \leq x \leq 10$, on your graphics calculator and copy the graph into your answer booklet.
- b) Use your calculator to determine, to the nearest unit, how many sets must be produced and sold to yield a profit of \$21,000.
- c) Use your calculator to determine how many sets must be produced and sold to yield the maximum possible profit.

2. Find the *exact* value of each of the following limits. Use calculus, not your calculator :

a) $\lim_{x \rightarrow 3} \frac{2x^2 - 6x}{x^2 + x - 12}$

b) $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3}$

c) $\lim_{x \rightarrow \infty} \frac{2x^3 + 3x^2 - 5x + 1}{3x^3 - 7x^2 + x - 5}$

3. Find the slope of the line tangent to each of the following curves at the point where $x = 1$:

a) $y = (x^2 + 3x + 2)(x^4 - 2x^3 + 2x + 3)$

d) $y = \sqrt{x^2 + 8}$

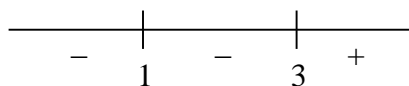
b) $y = \frac{x^2 - 3x - 5}{x^2 + 2x - 2}$

e) $y = e^{x^3}$

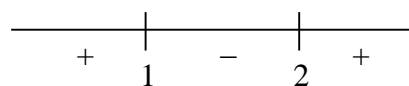
c) $y = (x^2 - 3x + 3)^3$

4. Given that $f(1) = 5$ and $f(3) = -2$ sketch the graph a continuous function f which satisfies the following conditions concerning the signs of its derivatives:

$f'(x)$



$f''(x)$



(continued on other side)

5. The cost of manufacturing q units of a commodity is $C = \frac{1}{2}q^2 + 2q + 1000$ dollars. A machine can produce a quantity of $q = t^2 + 10t - 4$ items during the first t hours of production.
- At what rate is manufacturing cost changing with respect to quantity 2 hours after production begins?
 - At what rate is manufacturing cost changing with respect to time 2 hours after production begins?

(Include proper units with your numerical answer.)

6. When the price of a certain commodity is p dollars per unit, consumers are willing to purchase x thousand units where $p^3 + xp^2 = 20$. If p increases at the rate of 1 dollar per month, at what rate is x changing when the price is \$2 per unit? Include proper units with your numerical answer.
7. Two concentric circles have radii 5 inches and 5.1 inches. Use approximation by increments to estimate the area between them. Express your answer to the nearest hundredth of a square inch.

8. Use calculus for this problem – calculator solutions are unacceptable.

A closed box having a square base and a volume of 150 ft^3 is to be constructed out of material which costs 6 cents per square foot for the top and bottom and 5 cents per square foot for the sides. Let x be the length and width of its base dimension and let y be its height.

- Express the cost of the box as a function of x .
 - Determine the dimensions of the box (length, width and height) which minimize the cost of the box.
 - What is the minimum cost?
9. Money is invested at an annual interest rate of 8% compounded continuously. (Recall that $B = Pe^{rt}$ for continuous compounding of interest.)
- How much will you have at the end of 2 years if you invest \$800 today?
 - How much do you have to invest today in order to have \$5000 after 3 years?
 - How long will it take for your money to triple?
 - If you invest \$1000 today, how fast is your money growing at the end of one year?

(Round your answers to the nearest penny.)