QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2.5 hours

Mathematics 131   **Fall 2016**

INSTRUCTIONS: ANSWER ALL QUESTIONS   SHOW ALL WORK

1.   Given $f(x) = 2x^2 + 4x - 3$, use **the definition of the derivative** to find $f'(x)$.

2.   Evaluate each limit:

   a) $\lim\limits_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$

   b) $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x^2 - 4}$

   c) $\lim\limits_{x \to \infty} \dfrac{2x^3 - 3x^2 - 5}{5x^3 + 2x^4 + 1}$

3.   Find the derivative for each of the following functions:

   a) $f(x) = \dfrac{x^4}{4} + \dfrac{2}{x^2} - \dfrac{3}{\sqrt{x}}$

   b) $x^3 y^2 - 4x^3 + y = 16$

   c) $y = 3e^{4x} \ln x^3$

   d)   Use logarithmic differentiation: $f(x) = \dfrac{e^{x^2} \sqrt{3x + 5}}{(2x - 3)^3}$

4.   If $f(x) = x^3 + 3x^2 + 1$, use calculus to find where the graph of f is increasing, where it is decreasing, where it is concave up and where it is concave down. Find any and all relative extrema and inflection points and sketch the graph using all the data.

5.   The total cost of producing x units is $C(x) = \frac{3}{2}x^2 + 12x + 1000$ dollars
   a)   Use marginal analysis to estimate the cost of the 15th unit.
   b)   Find the actual cost of the 15th unit.
   c)   If the current level of production is 15 units, use the approximation by increments formula to estimate how the total cost will change if 18 units are produced.

6.   A publisher can produce a book at a cost of $ 4 per book. 20,000 books can be sold at $ 16 per book and for each $ 1 reduction in price, 2000 more books will be sold. Use calculus to find the price at which the books should be sold to maximize profit?

7.   a) How much money should be invested today at 3.5 % compounded quarterly so that it will be worth $20,000 in 10 years?

   b) $ 6500 is invested at an annual interest rate of 4 %. Compute the balance after 5 years if interest is compounded continuously.

8.   The consumer demand, x, and the unit price, p, in dollars for a certain product are related by the equation $0.03x^3 + 0.4xp + 3.8p^2 = 1200$. At the time when $p = 100$ and $x = 200$, p is increasing at the rate of $\$0.30$ per month. At what rate is the demand, x, changing?

9.   a) Find the absolute maximum and absolute minimum values of the function
$f(x) = x^4 - 8x^2 + 2$ on the interval $-3 \le x \le 1$.

b) Use the Intermediate Value Theorem to show there is at least one root of the equation $3x^3 - 4x^2 - 6x - 5 = 0$ that lies between 2 and 3.

10.   a) Solve for x, correct to the nearest hundredth: $3^{2x} = 19$.

b) Solve for x: $\log_3(4x - 1) = 5$.