MATHEMATICS 131   **FALL 2017**
INSTRUCTIONS: SHOW ALL WORK IN YOUR BLUE BOOK FOR ALL QUESTIONS.

1.   Find the limit. ($+\infty$, $-\infty$, and DNE are possible answers.)

   a) $\lim\limits_{x \to 3} \dfrac{x^2 - 6x + 9}{x^2 - 9}$

   b) $\lim\limits_{x \to 9} \dfrac{3 - \sqrt{x}}{x - 9}$

   c) $\lim\limits_{x \to -\infty} \dfrac{4x^2 + 7}{2x^3 - 5x - 1}$

   d) $\lim\limits_{x \to -\infty} \dfrac{x^4 - 7}{x^3 + 9}$

2.   Use the Intermediate Value Theorem to show that $f(x) = x^3 - 2x - 2$ has a zero on the interval (1,2).

3.   <u>Use the definition of the derivative</u> to find $f'(x)$ if $f(x) = \dfrac{5}{x}$.

4.   Differentiate the following functions. You need not simplify.

   a) $y = 3\sqrt{x} - \dfrac{1}{\sqrt{x^3}} + \dfrac{5}{x^2} + 2e$

   b) $y = \sqrt{x}(x^2 - 1)^3$

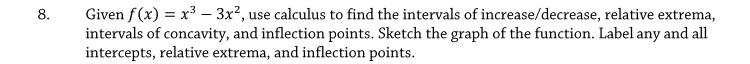   c) $y = 5e^x \ln x$

   d) $y = 7^x$ (Use logarithmic differentiation.)

5.   A ball is thrown straight up in the air from the roof of a building. The height of the ball at time $t \geq 0$ is $s(t) = -16t^2 + 96t + 112$, where s is in feet and t is in seconds.
   a)   Find the velocity of the ball one second after it is thrown.
   b)   Find the acceleration of the ball one second after it is thrown.

6.   A manufacturer's total monthly revenue is $R(q) = 150q - 0.04q^2$ dollars where q units are produced and sold during the month. Currently, the manufacturer is producing 50 units a month and is planning to increase the monthly output by 1 unit. Use marginal analysis to estimate the additional revenue generated by the production and sale of the 51st unit.

7.   Find an equation of the line tangent to the curve $3xy + y^2 = 4x^2 + 9$ at the point (0,3).

8. Given $f(x) = x^3 - 3x^2$, use calculus to find the intervals of increase/decrease, relative extrema, intervals of concavity, and inflection points. Sketch the graph of the function. Label any and all intercepts, relative extrema, and inflection points.

9. Find the absolute maximum and absolute minimum values of the function $f(x) = -3x^4 + 8x^3 - 10$ on the interval $0 \leq x \leq 3$.

10. A building has 250 apartments to rent. If x apartments are rented, the monthly profit is given by $p(x) = -8x^2 + 3200x - 80000$ dollars. How many apartments should be rented to maximize the building's profit?

11. How much money should be invested now at 8% compounded daily to have $ 25,000 in 4 years?

12. a) If $1,000 is invested at 7% compounded continuously, how long would it take for the investment to double?

    b) Would the doubling time change if the principal were something other than $1,000?