**QUEENS COLLEGE**
**DEPARTMENT OF MATHEMATICS**

**Final Examination**
**2 ½ Hours**

**Mathematics 131** **Fall 2018**

**Instructions: Answer all questions. Show all work.**

1. Given $f(x) = 3x^2 + 2x - 10$,
   a) use the definition of the derivative to find $f'(x)$
   b) find an equation of the line tangent to $f(x)$ at the point on the graph where $x = 1$.

2. Evaluate each limit, allowing ±∞ and DNE as answers.

   a) $\lim_{x \to -\infty} \frac{x^2+2x-8}{2x^3+4x^2}$ b) $\lim_{x \to 81} \frac{x-81}{\sqrt{x}-9}$ c) $\lim_{x \to 2} \frac{x^2+5x+6}{x^2-4}$ d) $\lim_{x \to 9} \frac{x^2+9}{x-9}$

3. Find the derivative for each of the following functions: (You do not need to simplify.)

   a) $h(x) = \frac{e^{x^3}\sqrt{x^2+x}}{(x-7)^{10}}$

   b) $g(x) = (x + 1)\sqrt{x^2 + 3}$

   c) $j(x) = \ln(x^6) + 5x^3 - \frac{1}{x^2} + \sqrt{x} + e$

   d) $k(x) = \frac{5-x^2}{2x+3}$.

4. a) \$3,200 is invested into a (clearly fictional) bank that pays an annual interest rate of 5.2%, compounded continuously.
   i) How much will the investment be worth in four years?
   ii) How long will it take for the value of the investment to double?

   b) How much money should be invested at an annual interest rate of 3% compounded monthly so that it will be worth \$2000 in five years?

5. Use the Intermediate Value Theorem to show that $f(x) = x^4 + 2x - 3$ has a zero on the interval (-2, -1).

6. Let $f(x) = 2x^3 - 3x^2 - 12x$. Use calculus to find
   a) i) the intervals of increase and intervals of decrease of $f$
   ii) the coordinates of all relative maxima and relative minima of $f$
   iii) the intervals of upward and downward concavity of $f$ and the coordinates of any inflection point(s)
   b) Using the results from part (a), sketch and label the graph of $f$. Be sure to clearly label all significant points.

**(continued on the back)**

7. Given $5x^3 - 2x^2y^3 = y^2 + 50x - 100$, find $\frac{dy}{dx}$.

8. Suppose a manufacturer's cost to produce x hundred widgets is given by the cost function C, where $C(x) = 0.25x^2 + 3x + 67$ dollars.
   a) Use marginal analysis to estimate the cost of producing the $4100^{th}$ widget.
   b) Find the actual cost of producing the $4100^{th}$ widget.

9. The quantity demanded each month of a manufacturer's product is related to the price per unit. The equation $p(x) = -0.0042x + 6$, where $p$ denotes the unit price and $x$ is the number of units demanded, relates the demand to price. The total monthly cost (in dollars) for producing $x$ units is given by $C(x) = 600 + 2x - 0.0002x^2$. Use calculus to determine how many units should be produced each month to maximize the manufacturer's monthly profit.

10. A carpenter has been asked to build a closed box with a square base and a volume of 2304 cubic meters. The material for the top and bottom of the box costs $1 per square meter, and the material for the sides costs $1.50 per square meter. Find the dimensions of the box that will minimize its cost of construction.

11. Consumer demand indicates that consumers buy $x$ units of a product each month when the price is p dollars per unit, where $x^2 + 5p^2 = 150$. Consumers buy 10 units when the price is $4 per unit. If the price is increasing at the rate of $0.50 per month, at what rate is quantity demanded $x$ changing per month?

**This material is the property of Queens College and may not be reproduced in whole or in part, for sale or free distribution, without the written consent of Queens College, Flushing, New York 11367.**