Queens College
Department of Mathematics
Final Examination
$2\frac{1}{2}$ HOURS

Mathematics 131   Spring 2015

Instructions: Answer all questions. Show all work in the exam booklet.

1.   Use the definition of the derivative to find $f'(x)$ for $f(x) = 2x^2 - 5x + 7$.

2.   Evaluate each limit. Indicate $+\infty$ or $-\infty$ if appropriate.

(a) $\lim\limits_{x \to 7} \dfrac{-x^2 + 10x - 21}{x^2 - 49}$

(b) $\lim\limits_{x \to +\infty} \dfrac{2x - 8x^3 + 6x^5}{4x^4 + x^3}$

(c) $\lim\limits_{x \to 0} \dfrac{\sqrt{x^2 + 16} - 4}{x^2}$

3.   Find the derivative of each of the following functions. Do NOT simplify your answers, but show all work.

(a) $f(x) = x^{-1} + 2x^{-\frac{1}{2}} - 3x^{-\frac{1}{3}} - 4^{-\frac{1}{4}}$

(b) $g(x) = \left(e^{x^3} + x^2\right)(8x^7 - x^4)$

(c) $y = \dfrac{(9x^6 + 4)(x - 3x^2)^7}{2x + 8}$ (Use logarithmic differentiation)

(d) $h(x) = \ln\left(\dfrac{2x^2 + 9}{1 - x^5}\right)$

4.   Let $f(x) = x^3 - 6x^2 + 11$.

(a)   Use calculus to find:
   i.   The interval(s) over which the graph of $f(x)$ is increasing and the interval(s) over which the graph of $f(x)$ is decreasing.
   ii.   The coordinates of all relative maxima and relative minima of $f(x)$.
   iii.   The intervals of upward and downward concavity of $f(x)$ and all inflection points of $f(x)$.

(b)   Using the result found in part (a), sketch the graph of $f(x)$.

5.   Given $2x^2 - y^2 = 3x^2y^3$, find $\dfrac{dy}{dx}$.

(continued on the back)

6. A manufacturer's total cost of producing t thousand units of a certain commodity is C hundred dollars, where C is given by $C = C(t) = \frac{1}{6}t^3 + 648t$.
   (a) Use marginal analysis to estimate the cost of producing the $13,000^{th}$ unit.
   (b) Find the actual cost of producing the $13,000^{th}$ unit.
   (c) If the current level of production is 8,000 units, estimate (using approximation by increments) how the total cost will change if 8,025 units are produced.

7. A company can produce video games at 22 dollars per game. Currently, when it charges 60 dollars per game, it sells 550 games a week. After an analysis of sales trends, it discovers that for every one dollar decrease in the selling price, it will sell 25 more games per week. Express the company's profit as a function of the number of one dollar decreases in the selling price. Then estimate the selling price that will maximize the company's profit.

8. A carpenter is offered 850 dollars to construct an open box with a square base that is to have a volume of 3,456 cm^3. The material used to construct the base costs 2 dollars per square centimeter and the material used to construct the sides costs 50 cents per square centimeter. Is it possible for the carpenter to turn a profit on this job? (*Hint: If the carpenter hopes to make a profit, he should minimize the cost of producing such a box.*)

9. A spherical soccer ball is punctured and its volume is deflating at a rate of 5.547π in^3/sec. If the radius of the soccer ball is currently decreasing at a rate of .075 in/sec, what is the radius of the ball? (*The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.*)

10. Walter invests 2,000 dollars at a fixed interest rate which is compounded continuously. After 8 years, his account grows to 3,500 dollars. How much money will Walter have in his account after 16 years? (*Hint: You don't need to explicitly solve for the interest rate in order to answer this question.*)