QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2.5 hours

Mathematics 131 **Spring 2016**

INSTRUCTIONS: ANSWER ALL QUESTIONS SHOW ALL WORK

1. Evaluate each limit. Indicate $+\infty$ or $-\infty$ or does not exist, where appropriate.
   a) $\lim_{t \to 2} \dfrac{t^3 - 4t}{t^2 - 2t}$
   b) $\lim_{x \to -\infty} \dfrac{x^4 + 1}{x^3 - 1}$
   c) $\lim_{x \to 9} \dfrac{3 - \sqrt{x}}{9 - x}$
   d) $\lim_{x \to 1} \dfrac{2x - 2}{x^3 + x^2 - 2x}$
   e) $\lim_{x \to 1} \dfrac{2x}{x - 1}$

2. Let $f(x) = -x^2 - 2x + 3$.
   a) Using the definition of the derivative, find f', the derivative of f.
   b) Find the slope of the tangent line to the graph of f at the point $(0, 3)$.
   c) Find an equation of the tangent line to the graph of f at the point $(0, 3)$.

3. A man wishes to have an enclosed rectangular vegetable garden in his backyard. If the garden is to have an area of 300 ft^2, find the dimensions that will minimize the amount of fencing needed for the garden.

4. Solve the given equations for x. Round your answers to the nearest thousandth where applicable.
   a) $\dfrac{5}{1 + 2e^{-x}} = 3$
   b) $\ln\left(\dfrac{x^3}{\sqrt{x}}\right) = 5$

5. Find the derivative of each of the following functions. (Algebraic simplifications are not required.)
   a) $f(x) = -\dfrac{1}{3}(x^{-3} - x^4)$
   b) $x^2y^3 - y^2 + xy - 1 = 0$
   c) $g(t) = \ln\left(t^2 e^{-t^2}\right)$
   d) $F(x) = \dfrac{e^{-x}}{x + \ln x}$
   e) $h(x) = \left(\dfrac{4x^{-1}}{x^3}\right)^3$
   f) $G(x) = \dfrac{2x}{(x^2 + 1)^2}$
   g) $p(x) = \dfrac{(x - 1)^3(x^3 + 1)^5}{(x^4 - 3)^9}$ (Hint: Use Logarithmic Differentiation)

(continued on the back)

6. Let $f(x) = x^4 - 2x^2$.
   a) Find the intervals where the function f is increasing and those where it is decreasing.
   b) Find the relative extrema of f.
   c) Find the intervals where the graph of f is concave upward and those where it is concave downward.
   d) Find the inflection points, if any, of f.
   e) Using the information found in parts a) – d), sketch the graph of $y = f(x)$. Indicate all important points.

7. Suppose the relationship between the unit price, "p", in dollars and the quantity demanded, "x", of the Acrosonic Model F loudspeaker system is given by the equation

$$p = -0.02x + 400 \quad (0 \le x \le 20{,}000)$$

The cost of producing "x" units of the loudspeaker is given by $C(x) = 100x + 200{,}000$ dollars.
   a) Find $R(x)$,the revenue function.
   b) Find $R'(x)$, the marginal revenue function.
   c) Find $P(x)$, the profit function.
   d) Find $P'(x)$, the marginal profit function.
   e) Compute $P'(2000)$ and interpret your result.

8. How long will it take $ 5000 to grow to $ 6500 if the investment earns interest at the rate of 6 percent per year compounded monthly?

9. A spectator standing at a distance of 4000 feet from a launch pad, observes a rocket being launched. If the rocket lifts off vertically and is rising at a speed of 600 feet/second when it is at an altitude of 3000 feet, how fast is the distance between the rocket and the spectator changing at that instant?