

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2.5 Hours

Mathematics 131

Spring 2017

Instructions:

Answer all questions.

Show all work.

1. Compute the following limits. ($+\infty$, $-\infty$, and DNE are acceptable answers.)

(a) (4 points) $\lim_{x \rightarrow +\infty} \frac{x^2 + x - 2}{x - 1}$

(b) (4 points) $\lim_{x \rightarrow 1} \frac{x}{(1 - x)^2}$

(c) (4 points) $\lim_{x \rightarrow 7} \frac{\sqrt{x + 2} - 3}{x - 7}$

(d) (4 points) $\lim_{x \rightarrow 4} \frac{x(x - 3)}{4 - x^2}$

2. (6 points) Use your calculator to make a table of values estimating $\lim_{x \rightarrow 0} \frac{\sqrt[3]{x + 8} - 2}{x}$ to four decimal places. Copy the table into your exam booklet.

3. (8 points) Find the derivative of $f(x) = \frac{2}{x}$ using the **definition** of the derivative.

4. Compute $\frac{dy}{dx}$ for the following functions. (You need not simplify.)

(a) (6 points) $y = 6x^5 - 5x^3 + x - 2 + e$

(b) (6 points) $y = \frac{1}{\sqrt{x^2 + 1}}$

(c) (6 points) $y = \frac{2 - x}{3x^2 + 1}$

(d) (6 points) $y = (x + 1) \ln(x^4)$

5. (8 points) Find an equation for the tangent line to the curve $x^2y - xy^2 + 4 = 2x + 2y$ at the point $(1, 1)$.

6. (8 points) At a certain factory, the daily output is $Q(L) = 200\sqrt{L}$ units, where L is the size of the labor force, measured in worker-hours. Currently 900 worker-hours of labor are used each day. *Use calculus to estimate* the effect on output that would be caused by reducing the labor force to 895 worker-hours.

7. (12 points) For the function $f(x) = \frac{x - 2}{x + 4}$, find, if any, all vertical and horizontal asymptotes, intercepts, intervals of increase/decrease, relative maxima/minima, intervals of concavity, and inflection points. Use this information to sketch the graph of $f(x)$. You may use your graphing calculator to assist with your sketch, but your sketch **MUST** be consistent with your calculations.

8. (8 points) Find the absolute maximum and minimum values of the function $f(x) = x^2e^{-x/2}$ over the interval $0 \leq x \leq 10$.

9. If you invest \$10000 now at an annual rate of 5%, how long will it take for your investment to grow to \$25000 if interest is compounded:

(a) (5 points) monthly?

(b) (5 points) continuously?

Round your answers to the nearest tenth of a year.

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