

**QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS**

Final Examination

$2\frac{1}{2}$ Hours

Mathematics 131

Spring 2019

Instructions: Answer all the questions. Show all work.

1. Let $f(x) = x^2 - 5x + 4$. Use the definition of the derivative to find $f'(x)$. Then find an equation of the tangent line to the graph of $y = f(x)$ at point with x -coordinate 6.

2. Evaluate each limit. Indicate $+\infty$ or $-\infty$ or does not exist where appropriate.
 - a) $\lim_{x \rightarrow 9} \frac{9 - x}{3 - \sqrt{x}}$
 - b) $\lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x^2 - 4x - 12}$
 - c) $\lim_{x \rightarrow \infty} \frac{3x^3 - 2x^2 + 4}{2x^3 + 3x^2 + 1}$
 - d) $\lim_{x \rightarrow 4^-} \frac{x^2 + 16}{x - 4}$

3. In each of the following, find y' :
 - a) $y = \frac{3x^2 + 2}{2x - 4}$
 - b) $y = e^{x^2} + \frac{2}{x^3} - \frac{4}{\sqrt{x}}$
 - c) $y^3 + 5x^2y = 6x - 2y + 5$
 - d) $y = \frac{e^{4x}(2x + 3)^3}{\sqrt{3x^2 - 4x}}$ (hint: use logarithmic differentiation)

4. Let $f(x) = x^3 + 3x^2 - 3$. Use calculus to determine intervals on which the graph of f is increasing, those on which it is decreasing, those on which it is concave upward, and those on which it is concave downward. Identify any and all relative maxima, relative minima and inflection points of the graph. Then use your results to sketch the graph of $y = f(x)$.

(continued on the back)

5. The total cost of manufacturing q units is $C(q) = \frac{1}{3}q^3 + 4q^2 + 20q$ dollars.
- Use marginal analysis to estimate the cost of the 25th unit.
 - Find the actual cost of the 25th unit.
 - Answer either (1) or (2). DO NOT DO BOTH.
 - If the current level of production is 25 units, use the approximation by increments formula to estimate how the total cost will change if 28 units are produced.
 - The demand, x units, and unit price, p , are related by the equation $x = 240 - 2p$. Express the elasticity of demand as a function of p . Calculate the elasticity of demand when $p = 100$ and $p = 50$. Interpret your answers for each one.
6.
 - How much money should be invested today at 4% compounded quarterly so that it will be worth \$ 15,000 in 10 years?
 - If \$ 3,500 is invested at an annual interest rate of 3.25%, what is the balance in 6 years if the interest is compounded continuously?
7. A store sold calculators at \$50 each and consumers bought 1500 each month. If it raises the price, it estimates that for each \$1 increase in price, 30 fewer calculators will be sold each month. Given that the store pays \$25 for each calculator it stocks, use calculus to determine the price at which a calculator should be sold in order to maximize the store's profit.
8. A 26 ft. ladder leaning against a wall starts to slide down. How fast is the distance from the top of the ladder to the base of the wall decreasing when the distance from the bottom of the ladder to the wall is 10 ft and the bottom of the ladder is sliding away from the wall at the rate of $\frac{1}{2}$ ft/minute?
9.
 - Find the absolute maximum and absolute minimum (if any) of the function $f(x) = \frac{1}{3}x^3 + 3x^2 - 12$ on the interval $-3 \leq x \leq 3$.
 - Using the function in part a), use the Intermediate Value Theorem to show that there is a root of the equation $f(x) = 0$ on the interval $[-3,0]$.