Mathematics 131 $\qquad$ **Spring 2022**

Instructions: Answer all questions.   Show all work.

1. Given $f(x) = \frac{2}{x}$.
   a) Use the definition of the derivative to find $f'(x)$.
   b) Find an equation of the line tangent to the graph of $f(x)$ at the point where $x = 1$.

2. Evaluate each limit, allowing $\pm\infty$ and DNE as answers.
   a) $\lim\limits_{x \to 1} \dfrac{x^2 + 4x - 5}{x^2 - 1}$

   b) $\lim\limits_{x \to 36} \dfrac{6 - \sqrt{x}}{36 - x}$

   c) $\lim\limits_{x \to +\infty} \dfrac{2x^5 - 3x}{4x^3 + 11}$

   d) $\lim\limits_{x \to 0^-} f(x)$, where $f(x) = \begin{cases} 2x + 10 & x < 0 \\ x^2 + 3x + 1 & x \geq 0 \end{cases}$

3. Find $\frac{dy}{dx}$ for each of the following. (You do not need to simplify.)
   a) $y = \dfrac{e^x \sqrt{x^2 - 2x}}{(5 - x)^7}$

   b) $y = 10e^{x^2} \ln x$

   c) $y = \ln(x^5) + \sqrt[3]{x^2} - \dfrac{1}{x^3} + x^e + 5x^{11} - \pi$

   d) $y = \dfrac{5 - x^4}{2x + 3x^2}$

   e) $y^3 + 7x^2 y = 6x - y + 10$

4. a) $4,200 is invested into a (clearly fictional) bank that pays an annual interest rate of 7.1%, compounded continuously. How long will it take for the value of the investment to triple?

   b) How much money should be invested at an annual interest rate of 2% compounded monthly so that it will be worth $1000 in six years?

5. Given $f(x) = x^3 + 3x^2 - 7$.
   a) Use the Intermediate Value Theorem to show that $f(x)$ has a zero on the interval $-1 \leq x \leq 2$.
   b) Find and classify the absolute extrema of $f(x)$ on the interval $-1 \leq x \leq 2$.

6. Let $f(x) = x^3 - 6x^2 + 5$. Use calculus to find
    a) (i)   the intervals of increase and intervals of decrease of f.
        (ii) the coordinates of all relative maxima and relative minima of f.
        (iii) the intervals of upward and downward concavity of f and the coordinates of any inflection point(s).
    b) Using the results from part (a), sketch and label the graph of f.

7. Suppose a manufacturer's cost to produce x snowboards is given by the cost function C, where $C(x) = 0.2x^2 + 10x + 1900$ dollars.
    a) Use marginal analysis to estimate the cost of producing the 501st snowboard.
    b) Find the actual cost of producing the 501st snowboard.
    c) If 500 snowboards are produced, find the average cost per unit.

8. Each month, the quantity demanded for a manufacturer's product is related to the price per unit. The equation $p(x) = -0.004x + 11$, where p denotes the unit price and x is the number of units demanded, relates the demand to price. The total monthly cost (in dollars) for producing x units is given by $C(x) = 500 + 2x - 0.001x^2$.

    Find equations for the manufacturer's monthly revenue and monthly profit. Then <u>use calculus</u> to determine how many units should be produced each month to maximize profit.

9. A stone dropped in a lake sends out a circular ripple whose radius increases at a constant rate of 3 ft/sec. When the radius is 15 feet, at what rate is the area of the circle enclosed by the ripple increasing?