

Queens College
Department of Mathematics

Mathematics 141

Final Examination

Fall 2015

Instructions: Answer *all* questions. *Show all work.* Justify your answers. If you use a calculator, show the commands that you used.

1. a) (4 points) Find the domain and the range of the function $p(x) = \sqrt{9 - x^2}$. Express your answer using interval notation.
b) (4 points) Let $F(x) = (\sin(x^3) + 9)^2$. Find functions f , g , and h such that F is the composition of f , g , and h (i.e., $F = f \circ g \circ h$).
2. Compute, *without using a calculator*, the following limits. (Real numbers, ∞ , and $-\infty$ are acceptable answers.)
 - a) (4 points) $\lim_{x \rightarrow 2} \frac{\sqrt{x^2+12}-4}{x-2}$
 - b) (4 points) $\lim_{x \rightarrow -2} \frac{|x|-2}{x^2+4}$
 - c) (4 points) $\lim_{x \rightarrow (\frac{\pi}{2})^+} \tan x$
 - d) (4 points) $\lim_{x \rightarrow -\infty} \frac{x^2-3}{\sqrt{4x^4+1}}$
3. (8 points) Use the table feature of your calculator to estimate $\lim_{x \rightarrow \infty} ((x^3 - 5x^2)^{1/3} - x + 1)$ to two decimal places. Copy four rows of the table into your exam booklet.
4. (10 points) *Using the definition of the derivative*, find $f'(2)$ if

$$f(x) = \frac{1}{x^2 + 1}.$$

5. (8 points) Compute the linearization of the function $f(x) = x^{3/4}$ at $a = 16$ and use it to approximate $(16.08)^{3/4}$.
6. Compute $\frac{dy}{dx}$ (you need not simplify) if:
 - a) (4 points) $x^3y + y^3x = 1$
 - b) (4 points) $y = \frac{1}{\sqrt{x^2-1}}$
 - c) (4 points) $y = \frac{\cos x + 1}{\sin^2 x}$
 - d) (4 points) $y = \left(\frac{x^2-2}{x^2+2}\right)^9$
7. a) (4 points) State the Intermediate Value Theorem.
b) (4 points) Use this theorem to prove that the equation $x^2 - \sin x - 1 = 0$ has at least one solution in the interval $(0, 2)$.

8. (10 points) Find the intervals of increase and decrease, and the intervals of concavity for the function

$$y = \frac{x^2 + 1}{x^2 - 4}.$$

Sketch the graph of the function, making sure to label all relative maxima and minima, inflection points, and asymptotes. You are encouraged to use your graphing calculator to help with your sketches, but the important features of the graph must be justified by calculus.

9. (8 points) If 432 in² of material is available to make a box with a square base and an open top, find the largest possible volume of the box. Make sure to show that you found the largest volume possible.
10. (8 points) A particle is moving along a hyperbola $xy = 36$. As it reaches the point $(4, 9)$, the x -coordinate is decreasing at a rate of 12 cm/s. How fast is the y -coordinate of the point changing at that instant? Is the particle moving up or down? Justify your answer.

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