MATHEMATICS 141   FALL 2016
INSTRUCTIONS: SHOW ALL WORK IN BLUE BOOKS FOR ALL QUESTIONS.

1.   Evaluate the following limits. (If a limit is ∞, $-\infty$, or DNE, show why without the use of calculator.)

   a) $\displaystyle\lim_{x \to 3} \frac{x^2 + 6x - 27}{x^2 + 4x - 21}$

   b) $\displaystyle\lim_{x \to \infty} \frac{x^2 - 3x^3 - x^5 - 18}{4x^3 - x - 42 + x^4}$

   c) $\displaystyle\lim_{x \to 10^+} \frac{|x - 10|}{-(x - 10)}$

   d) $\displaystyle\lim_{x \to \infty} \frac{5x - \sin x}{2x - 3}$

   e) $\displaystyle\lim_{x \to 4} \frac{\sqrt{x + 5} - 3}{x - 4}$

2.   Let $F(x) = \dfrac{3}{x}$. Using **_the definition of derivative_**, find $F'(x)$.

3.   For what value of the constant k is the function $f(x) = \begin{cases} k\cos(\pi x) + 1 & \text{if } x \le 4 \\ -x^2 + 2k & \text{if } x > 4 \end{cases}$ continuous everywhere?

4.   In each of the following, find $\dfrac{dy}{dx}$. (Algebraic simplification is not needed.)

   a) $y = -9x^8 + \dfrac{5}{x^4} + \sqrt[7]{x^9} - \pi^5$

   b) $y = (\sin^5 x - 10x)^9 (3x + 4)$

   c) $y = \dfrac{\tan(x^2)}{x^3 + 5x}$

   d) $\sin(x^2 + y) = x + y^3$

5.   Let $G(x) = \sqrt[3]{x}$.
   a)   Estimate the value of $G(8.02)$ using linear approximation or differentials.
   b)   Find an equation of the normal line to the graph of $G(x)$ at the point $(8, 2)$.

6.   The area of an expanding rectangle is increasing at the rate of 48 square centimeters per second. The length of the rectangle is always equal to the square of its width (in centimeters). At what rate is the width increasing at the instant when the width is 2 cm?

7.   a) Show that the function $f(x) = 6x^5 + 7x + 9$ has exactly one root in the interval $[-2, 0]$.
   b) Use your graphing calculator to estimate the root to three decimal places.

8.   Given $f(x) = \dfrac{x - 4}{x^3}$.
   a)   Find f' and f'' and simplify both.
   b)   Find all critical numbers of f.
   c)   Find the intervals on which f is increasing and those on which it is decreasing.
   d)   Find all local extrema of f.
   e)   Find the intervals where f is concave upward, where it is concave downward, and any inflection points of f.
   f)   Find any horizontal or vertical asymptotes of f.
   g)   Sketch the graph of f. (The graph should agree with your answers from parts a) to f).)

9.   Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$?