**QUEENS COLLEGE**
**DEPARTMENT OF MATHEMATICS**

**FINAL EXAMINATION**
**$2\frac{1}{2}$ HOURS**

**MATHEMATICS 141** **FALL 2017**
**INSTRUCTIONS: SHOW ALL WORK IN YOUR BLUE BOOK FOR ALL QUESTIONS.**

1. a) Compute the following limits, with $\infty$ and $-\infty$ as possible values:

i. $\lim_{x\to -2^+} \dfrac{x^2+x-2}{x^2+4x+4}$

ii. $\lim_{x\to 1} \dfrac{\frac{1}{3}-\frac{1}{x+2}}{x^2-1}$

b) Using the table feature of your calculator, either estimate the following limit to 4 decimal places or determine that it is $\infty$ or $-\infty$. Include at least the last 2 lines of your table as part of your answer.

$$\lim_{x\to 3} \frac{x^2-3x}{2^x-8}$$

2. Let $f(x)=\begin{cases} x+1, & \text{if } x\le 0 \\ x^2-1, & \text{if } 0<x<2 \\ 5-x, & \text{if } x\ge 2 \end{cases}$.

a) Draw the graph of $f$.
b) Is $f$ continuous at $x=2$? Explain.
c) Is $f$ continuous on $[0,1]$ ?
d) Is $f$ continuous on $(0,3)$ ?

3. Let $f(x)=\sqrt{4x+1}$
a) Using the definition of the derivative, find $f'(x)$.
b) Find an equation for the line which is tangent to the graph of $f$ at the point $(2,3)$.

4. In each case, find $\dfrac{dy}{dx}$ . (There is no need to simplify your answers.)

a) $y=\left(\dfrac{3}{x^4}+5\right)^6 \cos(7x+8)$

b) $y=\dfrac{\sec x}{\tan(x^2)}$

c) $x\sin y+y^3=5x-4$

5. At a certain instant, a dog is 4 ft. from the base of a telephone pole, running towards the pole at the rate of 20 ft./sec., while a squirrel is 3 ft. above the ground on the pole, running up the pole at the rate of 15 ft./sec. At this instant, at what rate is the distance between the dog and the squirrel changing? Is the distance between them increasing or decreasing?

6. a) Carefully state the Intermediate Value Theorem.
b) Use the Intermediate Value Theorem to show that $\dfrac{3c^3}{c^2+2}-c=3$ for some real number $c$.
c) Use your calculator to find the value of $c$, correct to 4 decimal places.

**(CONTINUED ON THE BACK)**

7. Let $f(x) = 12 + 4x^3 - x^4$.
   a) Find the intervals on which $f$ is increasing and the intervals on which $f$ is decreasing.
   b) Find the points (if any) where $f$ has a local minimum and the points (if any) where $f$ has a local maximum.
   c) Find the intervals on which the graph of $f$ is concave up and the intervals on which the graph of $f$ is concave down.
   d) Find the coordinates of all inflection points of the graph of $f$.
   e) Using the information found in parts a) – d), sketch the graph of $f$.

8. A rectangular enclosure with an area of 100 sq. yards is to be constructed with fencing that costs $ 2/yard on the outside and then divided into two equal rectangular pens with fencing that costs $5/yard in the inside.
   a) What is the smallest possible cost of the project if there is an unlimited amount available of each kind of fencing?
   b) What is the smallest possible cost of the project if there is an unlimited amount available of the cheaper fencing but only 4 yards available of the more expensive fencing?

9. Draw the graph of a single function $f$ having <u>all</u> of the following properties:
   i) $f'(x) > 0$ if $x < -4$ or $x > 2$
      $f'(x) < 0$ if $-4 < x < -2$ or $-2 < x < 2$

   ii) $f''(x) > 0$ if $-2 < x < 4$
      $f''(x) < 0$ if $x < -2$ or $x > 4$

   iii) $\lim_{x \to -2^-} f(x) = -\infty$, $\lim_{x \to -2^+} f(x) = \infty$

   iv) $\lim_{x \to \infty} f(x) = -1$

   v) $f(-4) = f(0) = 0$, $f(2) = -6$, and $f(4) = -3$