QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION
$2\frac{1}{2}$ HOURS

MATHEMATICS 141 **FALL 2018**
INSTRUCTIONS: SHOW ALL WORK FOR ALL QUESTIONS IN ORDER TO RECEIVE MAXIMUM CREDIT.

1) Use algebraic methods and limit laws (not your calculator) to find each of the following limits. If the limit is $+\infty$ or $-\infty$, explain why.

   a) $\lim_{x \to +\infty} \frac{5x^3 + 12x^2 + 9}{3x^3 - 5x}$

   b) $\lim_{x \to 5^+} \frac{2 - 3x}{x^2 - 2x - 15}$

   c) $\lim_{x \to 81} \frac{x - 81}{\sqrt{x} - 9}$

   d) $\lim_{x \to 0} x \csc(2x)$

2) Consider the function given by $f(x) = \frac{x^2 - x - 6}{x^2 - 9}$.
   a) Use the definition of continuity to explain why f is discontinuous at $x = 3$.
   b) Now consider the function given by $g(x) = \frac{x^2 - 6}{x^2 - 9}$. Is the discontinuity of g at $x = 3$ different from the discontinuity of f at $x = 3$? If so, how? If not, why?

   You may support your answer with an explanation and/or by drawing a reasonably accurate sketch of the graphs of f and g.

3) Let $F(x) = 2x^2 - 6x - 1$.
   a) Using the limit-based definition of the derivative, find $F'(x)$.
   b) Write an equation of the line which is tangent to the graph of F at the point on the graph where $x = -4$.

4) In each of the following, find $\frac{dy}{dx}$: (Algebraic simplification is not necessary)

   a) $y = \left(3x - \sqrt[3]{x}\right)\left(11\sqrt{x} + \frac{2}{3}x\right)$

   b) $y = \sqrt{\sin(\pi x) + \frac{1}{x}}$

   c) $y = \left(\frac{7x^3}{x^5 + 2x}\right)^5$

   d) $\cos(x^2) + 3y = x + 2x \tan(y)$

5) Find $f''(x)$ for the function given by $f(x) = \frac{1}{2}\sin^2 x - \sec 2x$.

6) A man uses a ladder that is 29 feet in length to climb onto the roof of his home to clean the leaves out of his gutters. As he throws the leaves off the roof, some of the leaves land in his neighbor's yard. Shortly thereafter, the disgruntled neighbor walks over to the ladder and pulls the base of the ladder away from the house at a rate of 2 feet per second. ☹

   How fast is the top of the ladder moving down the side of the house at the moment when the base of the ladder is exactly 20 feet from the house?

(CONTINUED ON THE BACK)

7) a) State the Mean Value Theorem.

b) Using the function given by $f(x) = \frac{x+1}{x}$ on the interval $\left[\frac{1}{3}, 3\right]$, find all values c that satisfy the conclusion of the Mean Value Theorem.

8) Given $g(x) = x^4 - 2x^3 - 6$,

a) i) find the intervals on which g is increasing and the intervals on which g is decreasing

ii) find the points on the graph of g (if any) where g has a local minimum and the points (if any) where g has a local maximum

iii) find the intervals on which the graph of g is concave up and the intervals on which the graph of g is concave down

iv) find the coordinates of any and all inflection points of the graph of g.

b) Using the information found in part a), draw a reasonable sketch the graph of g, and label all appropriate points.

9) A jewelry box with a square base is to be constructed with nickel-plated bottom and top and copper-plated sides. The box will have a volume of 40 cm^3.

If nickel plating costs \$2 per cm^2 and copper plating costs \$1 per cm^2, find the dimensions of the jewelry box that will minimize the cost of the materials used to construct the box. Round your answers to the nearest hundredth.