

**QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS**

Final Examination (2 hours 30 minutes)

Mathematics 141

Fall 2019

Instructions: Answer all the questions. Show all work.

1. Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$, $-\infty$ or does not exist (DNE), explain your reasoning.
 - a) $\lim_{x \rightarrow 1} \frac{4x}{x+2} - \frac{4}{3}$
 - b) $\lim_{x \rightarrow -2^+} \frac{x^2 - x - 6}{x^2 + 4x + 4}$
 - c) $\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x^2(1 + \cos x)}$
 - d) $\lim_{x \rightarrow -\infty} \cos\left(\frac{x^5 + 1}{x^6 + x^5 + 100}\right)$

2.
 - a) Let $f(x) = \sqrt{1 - 9x}$. Using the definition of the derivative, find $f'(x)$.
 - b) Find the domains of $f(x)$ and $f'(x)$.

3.
 - a) State the Intermediate Value Theorem.
 - b) Use the Intermediate Value Theorem to show that $f(x) = \frac{1}{16}x^4 - x^3 + 3$ has a zero on the interval $(1,2)$.
 - c) Use your calculator to find the zero of $f(x)$ on $(1,2)$. Round your answer to 5 decimal places.

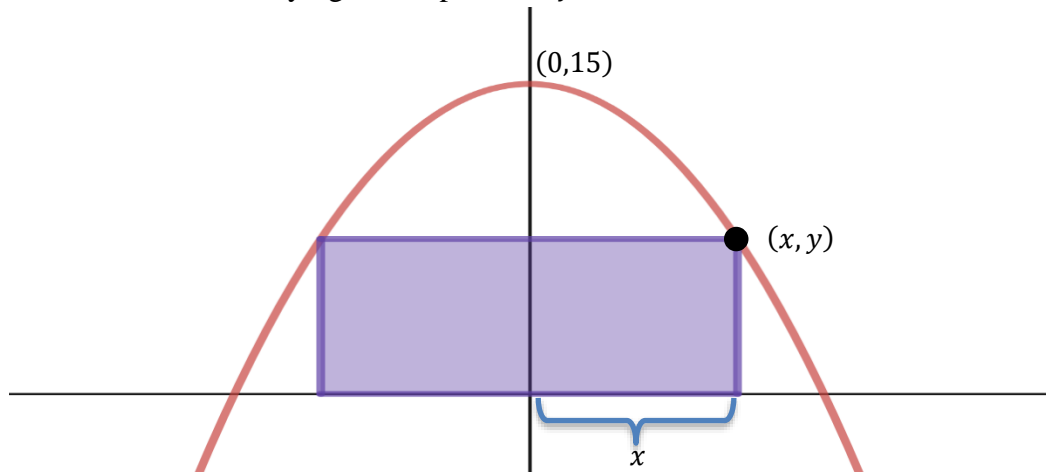
4. Use the method of linear approximation to estimate the value of $(7.94)^{2/3}$.

5. In each of the following, find $\frac{dy}{dx}$. (Algebraic simplification is not necessary.)
 - a) $y = \sqrt[3]{2 + \tan(x^2)}$
 - b) $y = (x^{-4} - 2)^3(5x^2 + 1)^{-2/3}$
 - c) $x = \sin(y^2) - 5x^4y$
 - d) $y = \left(\frac{\sec^3 x - 2\pi^6}{4x^5 + 3x^4 - x}\right)^8$

6. Let $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$.
 - a) Find the vertical and horizontal asymptotes of the graph of f , if any.
 - b) For which intervals is f increasing and for which intervals is f decreasing?
 - c) Find all the local maxima and/or local minima of f .
 - d) For which intervals is the graph of f concave up and for which is it concave down?
 - e) Find the inflection points of the graph of f , if any.
 - f) Sketch the graph of $y = f(x)$ using the information found in parts a) – e).

(continued on the back)

7. Find the dimensions of the rectangle of largest area that has its base on the x -axis and its other two vertices above the x -axis and lying on the parabola $y = 15 - x^2$.



8. What point on the curve $y = \sqrt{x + 4}$ comes closest to the origin?
9. The equation of motion of a particle moving along a horizontal line is $s(t) = 2t^3 - 7t^2 + 4t + 1$, where s is measured in meters and t is measured in seconds.
- Find the velocity and acceleration functions of the particle with respect to t .
 - Find the acceleration of the particle after one second.
 - At what time(s) does the particle come to rest?
 - When is the particle moving to the right?
10. Jessica is filling a conical paper cup with lemonade. The cup is 12cm tall and has a radius of 12 cm. If the level of the lemonade rises at a constant rate of 6 cm/s, how fast is the volume of the lemonade changing when the height of the lemonade is 8 cm? ($V_{cone} = \frac{1}{3}\pi r^2 h$)