

**QUEENS COLLEGE
MATHEMATICS DEPARTMENT**

**FINAL EXAM
2 ½ HOURS**

Mathematics 141

Spring 2015

INSTRUCTIONS:

ANSWER ALL QUESTIONS

SHOW ALL WORK

1. Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$, $-\infty$, or does not exist, explain why.

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 10x + 21}{x^2 - 9}$

(b) $\lim_{x \rightarrow 5^+} \frac{\sqrt{2x-1}-3}{x-5}$

(c) $\lim_{\theta \rightarrow 0} \frac{\sin 5\theta}{6\theta + \tan 2\theta}$

(d) $\lim_{x \rightarrow -6} \frac{4x+24}{|x+6|}$

(e) $\lim_{x \rightarrow -\infty} \frac{5x^4 + 7x^2 + 1}{3x^3 + 8x}$

2. Use the TABLE menu of your calculator to find a three decimal place estimate of $\lim_{x \rightarrow 3} \frac{x^2 - 9}{2^x - x - 5}$. Copy the resulting table into your exam booklet, displaying enough values to justify your answer.

3. Let $f(x) = \frac{x}{3x+1}$. Using the definition of the derivative, find $f'(x)$.

4. In each of the following, find $\frac{dy}{dx}$. (You need not simplify.)

(a) $y = (x^5 - 8x^2 + 4)^6 (9x^3 - 2x^2 + 1)^5$

(b) $y = \frac{\sec 5\theta}{1 + \tan 6\theta}$

(c) $y = \cos(\tan \sqrt{\sin 4x})$

(d) $\sin(x + y) = y^3 \cos x$

5. (a) Show that the equation $3x + \cos x - 2 = 0$ has exactly one real root. Justify your conclusion.
(b) Use your graphing calculator to find the root in part (a), correct to three decimal places.

(continued on the back)

6. Use the method of linear approximation or differentials to estimate $\sqrt[6]{63.96}$.
7. Let $f(x) = \frac{3x}{x^2-4}$.
- (a) For which intervals is f increasing and for which is f decreasing?
 - (b) Find all the local maxima and/or local minima of f .
 - (c) Find the vertical and horizontal asymptotes of the graph of f , if any.
 - (d) For which intervals is the graph of f concave up and for which is it concave down?
 - (e) Find the inflection points of the graph of f , if any.
 - (f) Sketch the graph of $y = f(x)$ using the information found in parts (a) – (e).
8. A cone-shaped paper cup is being filled with water at the rate of 3 cubic centimeters per second. If the height of the cup is 10 centimeters and the radius of its base is 6 centimeters, how fast is the water level rising when the level is 5 centimeters?
(Hint: The volume of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)
9. Find the dimensions of the rectangle of largest area that has its base on the x -axis and its other two vertices above the x -axis and lying on the parabola $y = 8 - x^2$. Make sure to justify that your answer gives a maximum value.