QUEENS COLLEGE
MATHEMATICS DEPARTMENT

FINAL EXAM
2 ½ HOURS

Math 141 **Spring 2016**

INSTRUCTIONS: **ANSWER ALL QUESTIONS** **SHOW ALL WORK**

1) Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$ or $-\infty$ or does not exist, explain why.

   a) $\lim\limits_{x\to -3^+} \dfrac{x^2+5x+6}{x^2+6x+9}$

   b) $\lim\limits_{x\to 4} \dfrac{\frac{2x}{x+3}-\frac{8}{7}}{x-4}$

   c) $\lim\limits_{x\to -\infty} \dfrac{1-2x-7x^3}{5x^3-4}$

2) Use your calculator to approximate $\lim\limits_{x\to 0^-} (x+1)^{\frac{1}{x}}$. Construct an appropriate table, copy it into your booklet, and then use it to estimate the indicated limit with four-decimal-place accuracy. Include at least six appropriately chosen $x-$values.

3) Use the definition of the derivative to find $f'(x)$ when $f(x)=\sqrt{2x-3}$.

4) a) Carefully define what it means when we say that "function f is continuous at $x = a$."

   b) Let $f(x)=\begin{cases} x^3-16 & \text{if } x<3 \\ 4x-1 & \text{if } 3< x<5 \\ x^2 & \text{if } x\geq 5 \end{cases}$

   Using only the definition in a) above, determine and explain clearly whether f is continuous

   (*i*) at $x=3$. (*ii*) at $x=5$.

5) Find $\frac{dy}{dx}$ for each of the following (algebraic simplification is not necessary):

   a) $y=5x^3+20\sqrt[4]{x^7}-\frac{2}{x^3}-4\pi^7-10x+11$

   b) $y=\sqrt[3]{\frac{\cot x}{1-\sec x}}$

   c) $y=\left(8\sqrt{x}-5\right)^3\left(x^2-7x+10\right)^5$

   d) $y=\tan^4\left(\sin\left(6x^7\right)\right)$

6) Find an equation of the line tangent to $x^2y^3+4\cos y=x^5-y+3$ at the point (1, 0).

(continued on other side)

7) Man A is 32 miles north of man B. At 6 PM, man A drives south at the rate of 40 miles per hour and man B bicycles east at 10 miles per hour. At 6:30 PM, will the distance between man A and man B be increasing or decreasing? At what rate? (Round answer to the nearest integer.)

8) Let $f(x) = 4x^3 - x^4 - 15$.
   a) Use calculus to find the intervals where f is increasing and the intervals where f is decreasing.
   b) Use calculus to find the local maximum and local minimum points of f, if any.
   c) Use calculus to find the intervals where the graph of f is concave up and the intervals where the graph of f is concave down.
   d) Sketch the graph of f. Label the local maximum and minimum points and the points of inflection, if any.

9) Show that the equation $\cos x = 2x - 8$ has at least one positive root, stating the theorem that you used. Then use your calculator to estimate this root, accurate to five decimal places.

10) A farmer wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. The total area of entire field must be exactly 2400 square feet. He also wants to divide the field into two pens by placing an additional fence perpendicular to the river. The fencing that will be used parallel to the river costs \$2 a foot, and the fencing that will be used perpendicular to the river costs \$4 a foot. What are the dimensions of the field that has the minimum cost? What is the minimum cost of this field? Make sure to show that you have found the minimum cost possible.