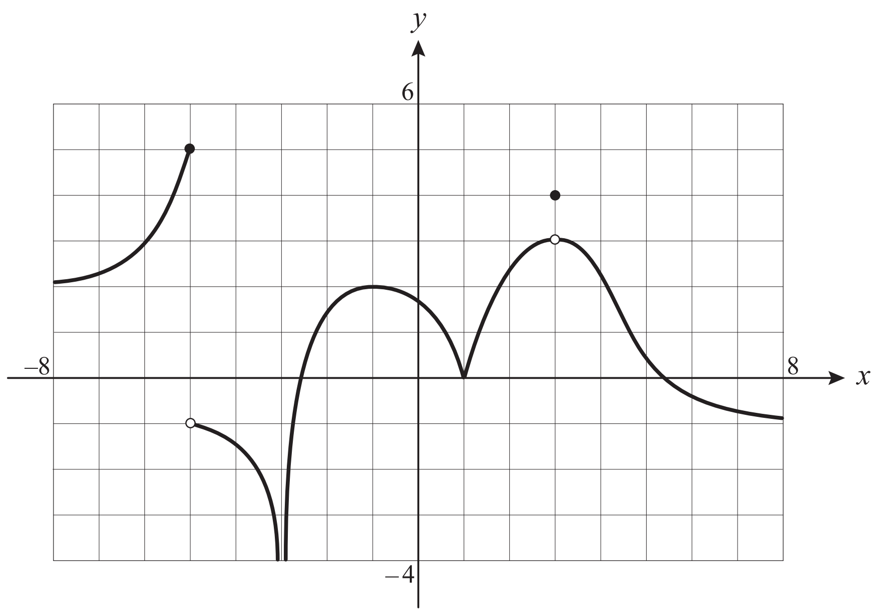
Department of Mathematics, Queens College
Math 141 Final Exam, Spring 2017

- **The exam has three parts.**
- **You have 150 minutes to answer the questions.**
- **Show all your work in your booklet.**

PART I [10 POINTS]. WARM-UP!

The graph of a function f on the interval $[-8, 8]$ is shown below:



Evaluate each of the following. If the result is $\pm\infty$ or does not exist, so state.

(i) $\lim\limits_{x\to-5} f(x)$ (ii) $\lim\limits_{x\to-3} f(x)$ (iii) $\lim\limits_{x\to1} f(x)$ (iv) $\lim\limits_{x\to3} f(x)$ (v) $f(3)$

(vi) $\lim\limits_{x\to+\infty} f(x)$ (vii) $\lim\limits_{x\to-\infty} f(x)$ (viii) $f'(-1)$ (ix) $f'(1)$ (x) $f'(3)$

PART II [32 POINTS]. CLEARLY WRITE THE LETTER OF THE CORRECT ANSWER IN YOUR BOOKLET. YOU MUST SHOW HOW YOU ARRIVE AT YOUR DECISION.

1. $\lim\limits_{x\to0} \dfrac{\sqrt{1-x}-1}{x}$

(A) is 0 (B) is $\dfrac{1}{2}$ (C) is $-\dfrac{1}{2}$ (D) does not exist

2. $\lim\limits_{x\to3^-} \dfrac{x-4}{x^2-9}$

(A) is $+\infty$ (B) is $-\infty$ (C) is 0 (D) does not exist

3. If the function $f(x) = \begin{cases} \dfrac{\sin(ax)}{x} & \text{for } x \neq 0 \\ 5a-1 & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then a is

(A) -4 (B) 4 (C) $-\dfrac{1}{4}$ (D) $\dfrac{1}{4}$

4. The limit $\lim\limits_{h\to0} \dfrac{\sqrt[3]{8+h}-2}{h}$ represents the derivative $f'(a)$, where

(A) $f(x) = \sqrt[3]{x},\ a = 2$ (B) $f(x) = \sqrt[3]{x},\ a = 8$
(C) $f(x) = \sqrt[3]{8+x},\ a = 2$ (D) $f(x) = \sqrt[3]{8+x},\ a = 8$

(continued on the other side)

5. Suppose f is a differentiable function such that $f(0) = 1$ and $f(10) = 1$. Then, there must be a point c in the interval $(0, 10)$ for which

(A) $f'(c) = 0$ (B) $f'(c) = 10$ (C) $f'(c) = \frac{1}{10}$ (D) $f'(c) = 9$

6. If ε is small, the best linear approximation to the volume of a cube of side-length $10 + \varepsilon$ is

(A) 1000 (B) $1000 + 100\,\varepsilon$ (C) $1000 + 200\,\varepsilon$ (D) $1000 + 300\,\varepsilon$

7. My calculator suggests that the absolute maximum of the function $f(x) = \dfrac{\sin x}{x^2 + 1}$ on the interval $(-\infty, \infty)$ is about

(A) 0.431 (B) 0.433 (C) 0.435 (D) 0.437

8. The position of an object moving along a straight line is given by $s = t^2 \cos t$, where s is measured in feet and the time t is measured in seconds. The *initial* acceleration of the object in ft/sec^2 is

(A) 2 (B) 1 (C) 0 (D) -1

PART III [58 POINTS]. SOLVE THE FOLLOWING 5 PROBLEMS.

Problem 1. [10 points] Show, only using appropriate theorems, that the equation

$$x^5 + 2x + 1 = 0$$

has a unique solution. Then use your calculator to estimate this solution to 3 decimal places.

Problem 2. [12 points] In each case, find the derivative $\dfrac{dy}{dx}$:

(i) $y = \dfrac{(x^2 - 1)^2}{x^3 + 1}$ (ii) $y = \dfrac{x}{2} + \dfrac{2}{x} + \tan(5x)$ (iii) $y^3 + \sin(xy) = x^2$

Problem 3. [10 points] A video display shows a right triangle whose base is *decreasing* at the rate of 0.2 cm/sec and whose height is *increasing* at the rate of 0.3 cm/sec. How fast is the area of this triangle changing at the moment when the base is 5 cm and the height is 12 cm? Is the area of the triangle increasing or decreasing at that moment?

Problem 4. [16 points] Consider the function $f(x) = x^4 - 16x^3 + 2000$.

(i) Write the formulas for f' and f'' and use them to find the critical point(s) and possible inflection point(s) of f.

(ii) Determine the intervals on which f is increasing or decreasing, and the intervals on which f is concave up or concave down. (You may simply put this information in a table if you wish.)

(iii) Classify each critical point of f found in (i) as a local maximum, local minimum, or neither. Also, list all the inflection points of f.

(iv) Use your findings in (i)-(iii) to sketch the graph of f.

Problem 5. [10 points] Use methods of calculus to find the coordinates of the point on the curve $y = x^{3/2}$ that is closest to the point $(20, 0)$.