QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 141 Spring 2018

Instructions. Answer each question in the blue book. Show your work and justify your answers.

1. Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$, $-\infty$, or does not exist, show why.
   (a) $\lim_{x\to 4} \dfrac{16-x^2}{2x^2-9x+4}$
   (b) $\lim_{x\to\infty} \dfrac{x^4}{(x^2+5)^2}$
   (c) $\lim_{x\to -3^+} \dfrac{x+7}{(x^2-9)}$
   (d) $\lim_{x\to -2} \dfrac{\sqrt{3x+7}-1}{x+2}$
   (e) $\lim_{x\to 0} \dfrac{\sin(5x)}{\sin(6x)}$

2. Find the derivative of $f(x) = x^2 + x + 1$ using the limit definition of a derivative.

3. Using the definition of continuity, determine if the function

$$h(x) = \begin{cases} 6x-8 & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ x^2 & \text{if } x > 2 \end{cases}$$

   is continuous at
   (a) $x = 2$
   (b) $x = 0$.

4. (a) Find the derivative of each of the following functions: (Algebraic simplification is unnecessary.)
   (i) $f(x) = (\frac{4x^2-1}{2x})^7$
   (ii) $g(x) = \sin^2(x^3 + 5x + \frac{1}{x})$
   (iii) $h(x) = 2x^4 \sec(3x) - \tan(\sqrt[3]{x^2})$
   (b) Find y'' if $y = \sqrt{4-2x^4+3x^8}$

5. If 2700 cm^2 of material is available to make a rectangular box with a square base and an open top, find the largest possible volume of the box.

6. A plane is flying directly away from you at 400 mph at an altitude of 3 miles. How fast is the plane's distance from you increasing at the moment when the plane is directly over a point on the ground 4 miles from you?

7. Consider the polynomial, $p(x) = x^5 + x - 1$
   (a) Use the Intermediate Value Theorem to show that there is at least one real root of $p(x)$ on the interval $[0, 1]$.
   (b) Now use Rolle's Theorem to show there is exactly one root on this interval.

8. Let $f(x) = \dfrac{4x}{x^2+1}$.
   (a) Determine
   (i) the domain of f.
   (ii) the critical numbers of f.
   (iii) the horizontal asymptotes of f, if any.
   (iv) the local minimum and maximum values of f.
   (v) where f is increasing and where f is decreasing.
   (vi) where f is concave up and where f is concave down.
   (b) Use the information found in part (a) to sketch the graph of f.