QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 141 Spring 2019

Instructions: **Answer all questions.** **Show all work.**

1. Using algebraic methods, find each of the following limits. If a limit is ∞, $-\infty$, or does not exist, state this as your answer.
   a) $\lim_{x\to 9^-} \dfrac{\sqrt{x}-3}{9x^2-x^3}$
   b) $\lim_{x\to -1} \dfrac{1-|x|}{1+x}$
   c) $\lim_{x\to \infty} \dfrac{x^4-x^3+5}{1-5x^4}$
   d) $\lim_{u\to 0} \dfrac{u^2\cdot \cot u}{\cos u \cdot \sin u}$

2. a) State the definition that a function $f(x)$ is continuous at $x=a$.
   b) Let
$$f(x)=\begin{cases} ax^2+b & \text{if } 0<x\le 2 \\ \frac{18}{x+1} & \text{if } x>2 \end{cases}$$
   If $f(1)=3$, determine the values of a and b for which $f(x)$ is continuous for all $x>0$.

3. a) State the definition of $f'(-1)$, the derivative of a function $f(x)$ at $x=-1$.
   b) Using only this definition, compute $f'(-1)$ for $f(x)=\frac{2}{x^2}$.

4. Find $\frac{dy}{dx}$ for each of the following. (Algebraic simplification unnecessary.)
   a) $y=(\cos 2x)\cdot(x^6-5x+3)$
   b) $y=\frac{2x+3}{\sqrt{x}+1}$
   c) $2y^2-x^4=9-xy$

5. Find the linear approximation of $g(z)=4\sqrt[5]{z}$ at $z=1$. Use the linear approximation to estimate the values of $4\sqrt[5]{1.5}$ and $4\sqrt[5]{.97}$. Compare these values to the ones found using your calculator. What conclusion can you draw?

6. Two people start from the same point. One bicycles west at 12 mi/h and the other jogs south at 5 mi/h. How fast is the distance between the people changing three hours after they leave their starting point?

7. Let $f(x)=x^3+6x^2-15x+1$. Use calculus to
   a) find the intervals of increase and intervals of decrease of f
   b) find the local maximum and minimum values of f, if any
   c) find the intervals where f is concave up and those where f is concave down
   d) find any and all inflection points of f.
   e) Use the information found in parts a) through d) to sketch the graph of $y=f(x)$.

8. Find the point on the graph of $f(x)=4-2x$ that is closest to the point $(1,1)$.

9. a) State the Mean Value Theorem.
   b) If f is a function such that $f(1)=k$, $f(3)=5$, and $f'(x)\le 1$ for all x, use the Mean Value Theorem to show that $k\ge 3$.