

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
FINAL EXAMINATION
 $2\frac{1}{2}$ HOURS

Mathematics 141

Spring 2022

Instructions: Answer all questions. Show all work.

1. Evaluate each limit. If a limit is $+\infty$, $-\infty$, or does not exist, explain why.

(a) $\lim_{x \rightarrow 49} \frac{7 - \sqrt{x}}{49 - x}$

(b) $\lim_{x \rightarrow 4^-} \frac{2x - 8}{|x - 4|}$

(c) $\lim_{x \rightarrow 3^+} \frac{x^2 - 9}{x^2 - 6x + 9}$

(d) $\lim_{x \rightarrow -1} \frac{\sin(x + 1)}{2x + 2}$

2. Show the following piecewise function f is continuous for all real values of x :

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ \sqrt{x + 1} & \text{if } x = 0 \\ \frac{1}{x + 1} & \text{if } x > 0 \end{cases}$$

3. (a) Use the definition of the derivative to find $f'(x)$ if $f(x) = \frac{1}{x + 2}$.

(b) Using the result found in part (a), find an equation of the tangent line to the graph of $y = f(x)$ at the point $(2, \frac{1}{4})$.

4. In each of the following, find $\frac{dy}{dx}$. (Algebraic simplification is unnecessary)

(a) $y = \frac{1}{x} + \sqrt[3]{x} - 7x^8 + 2\pi$

(b) $y = x^5 \sec^2 x$

(c) $y = \frac{\cos(3x - 4)}{\sin(4x^3)}$

(d) $x^4 + 4x^2y - y^3 = 4$

(continued on the back)

5. Two people are standing at the entrance to an elevator. At the same time as one person starts walking away from the elevator at a rate of 2 ft/s , the other person rides the elevator up at a rate of 3 ft/s . At what rate is the distance between the two people changing 4 seconds later? Interpret your answer.
6. Use linear approximation (i.e., differentials) to obtain an estimate for $\sqrt[4]{81.09}$.
7. Using appropriate theorems, show the equation $\frac{1}{2} \cos x + x - 1 = 0$ has exactly one root between 0 and $\frac{\pi}{2}$.
8. A bin for holding heavy material must be in shape of a rectangular box with a square base and open top. Material for the square base costs \$16 per square foot, while material for the four sides costs \$1 per square foot. If the volume of the bin must be 216 cubic feet, find the dimensions of the bin that will minimize its cost of construction.
9. Consider the graph of $y = f(x) = \frac{8x}{(x-2)^2}$.
- (a) Find the domain of f .
 - (b) Find the interval(s) on which f is increasing and those on which f is decreasing.
 - (c) Find and classify all local (relative) extrema of f .
 - (d) Find the interval(s) on which f is concave upward and those on which f is concave downward.
 - (e) Find the x -coordinates of any and all points of inflection of f .
 - (f) Find any and all horizontal and vertical asymptotes of the graph of f , justifying each by an appropriate limit.
 - (g) Based upon the results of parts (a)-(f), sketch the graph of $y = f(x)$, showing all important points.