Queens College
Department of Mathematics

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 142 **Fall 2016**

<u>Instructions</u>: Answer ALL Questions. Show ALL Work.

1. Evaluate the following integrals without the use of a calculator:

   i) $\displaystyle\int \tan x \ln(\cos x)\,dx$

   ii) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$

   iii) $\displaystyle\int_0^1 \frac{e^x}{1+e^{2x}}\,dx$

2. Using the definition of a definite integral as the limit of the Riemann sum, evaluate

$$\int_0^2 (2x - x^3)\,dx$$

   Hint:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

3. If $f(x) = \displaystyle\int_0^{\sin x} \sqrt{1+t^2}\,dt$ and $g(y) = \displaystyle\int_3^{y} f(x)\,dx$, find $g''(\pi/6)$.

4. If $f(x) = \ln x + \tan^{-1} x$, find $(f^{-1})'(\pi/4)$.

5. Find $\dfrac{dy}{dx}$:

   i) $y = e^{\cos x} + \cos(e^x)$

   ii) $y = \tan^{-1}\left(\sin^{-1}\sqrt{x}\right)$

   iii) $y = \ln(\sin x) - \dfrac{1}{2}\sin^2 x$

6. Use logarithmic differentiation to find $\frac{dy}{dx}$ if

$$y = \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}.$$

7. Find the exact length of the curve

$$y = \frac{x^4}{16} + \frac{1}{2x^2}, \quad \text{where } 1 \le x \le 2.$$

8. The region R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the line $x = -1$. Find the volume of the resulting solid.

9. Solve the initial-value problem

$$(1+\cos x)\frac{dy}{dx} = (1+e^{-y})\sin x, \quad \text{where } y(0) = 0.$$

10. The half-life of cesium-137 is 30 years. Suppose we have a 100-mg sample.

    i) Find the mass that remains after t years.

    ii) How much of the sample remains after 100 years?

    iii) After how long will only 1 mg remain?