Queens College
Department of Mathematics

Final Examination
$2\frac{1}{2}$ Hours

Mathematics 142 **Fall 2017**

<u>Instructions</u>: Show all work in your blue book for all questions.

1. Find y' for each of the following. Algebraic simplification is not needed.

   a) $y = \sec^2(e^x) + e^{-2x}$

   b) $y = \ln(x^2 + \ln x)$

   c) $y = \sin^{-1}(x^3) - \tan^{-1}(x^3)$

   d) $y = \int_{-x^2}^{2} e^t \cdot 10^{2t}\, dt$

   e) $y = \dfrac{(3x-5)^{10}(2x+3)^{-3}}{\tan^2 x}$

2. Let $f(x) = e^x + \tan^{-1} x + \pi - 1$.

   a) Show that $f(x)$ is one-to-one, hence has an inverse, $f^{-1}(x)$.

   b) Find $(f^{-1})'(\pi)$.

3. Use the definition of the definite integral as the limit of Riemann sums to evaluate $\int_0^3 (4x^3 - 6x^2 + 1)\, dx$.

   Note: $\sum_{i=1}^{n} i^3 = \dfrac{n^2(n+1)^2}{4}$; $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$

4. Find the following integrals:

   a) $\int \left(x^3 - x^{-2} + 6x^{3/4}\right) dx$

   b) $\int_{-1}^{1} x^3 \sin(3 + x^4)\, dx$

   c) $\int_0^2 \dfrac{e^x}{e^x + 10}\, dx$

   d) $\int \dfrac{\cos^{-1} x}{5\sqrt{1-x^2}}\, dx$

5. Let R be the region in the plane bounded by the curves $y = x^3 - 2x$ and $y = 2x$, where $x \geq 0$.

   a) Find the area of the region R.

   b) Find the volume of the solid of revolution obtained when R is rotated about the line $y = 4$.

   c) Find the volume of the solid of revolution obtained when R is rotated about the y-axis.

   d) Find the perimeter of the region R.

6. Solve the differential equation $\frac{dy}{dx} = (x^2 + 1)\cos^2 y$ when $x = 3$ and $y = \frac{\pi}{4}$.

7. Arsenic-74, used to locate brain tumors, has a half-life of 17.5 days. Suppose we have a sample of Arsenic-74 with a mass of 150 mg.

   a) Find a formula that computes the mass that remains after t days.

   b) How much of the sample of Arsenic-74 will remain after 10 days? (Round your answer to the nearest hundredth.)

   c) When will the sample have a mass of 30 mg? (Round your answer to the nearest hundredth.)