QUEENS COLLEGE
Department of Mathematics
Final Examination
$2\frac{1}{2}$ Hours

Mathematics 142 Fall 2018

Instructions. Answer each question. Show your work and justify your answers. Partial credit will be awarded for relevant work.

1. Find the derivative $\frac{dy}{dx}$ for each of the following functions. Please make the obvious simplifications.

a. $y = \int_1^{x^2} \sqrt{t^3 + t + 1}\, dt$ b. $y = \ln\left(\frac{(x-1)^3(x+4)^5}{\sqrt{x^2+x+1}}\right)$ c. $y = e^{x^2} \sin^{-1}(3x)$

2. Evaluate each of the following integrals by finding a suitable anti-derivative and applying the fundamental theorem of calculus. (Do not use the calculator for definite integrals.)

a. $\int_1^2 \frac{6x^4 - 5x + 4}{x^2}\, dx$ b. $\int \frac{e^{3x}}{9 + e^{3x}}\, dx$ c. $\int \frac{1}{1 + 9x^2}\, dx$

d. $\int_0^{\frac{\pi}{6}} \frac{\cos 3x}{(1 + \sin 3x)^2}\, dx$ e. $\int \frac{x}{\sqrt{9 - 4x^2}}\, dx$

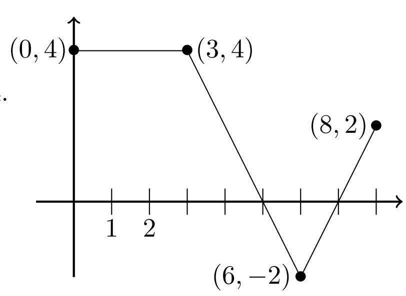
3. Find $\int_0^3 (x^2 + 1)\, dx$ as a limit of a Riemann sum. Include the following information:

a. Using sigma notation, write the Riemann sum with variable number n rectangles and suitable sample points of your choice. State clearly what you are using for step size, partition points and sample points.

b. Find the limit of the sum in part a as $n \to \infty$. Here are some useful formulas:

$\sum_{i=1}^{n} i = \frac{1}{2}n(n+1)$ $\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$ $\sum_{i=1}^{n} i^3 = \frac{1}{4}[n(n+1)]^2$

4.



The graph of $y = f(x)$ is shown here. Let $g(x) = \int_0^x f(t)\, dt$.

a. Determine the numerical value of $g(0)$, $g(5)$ and $g(7)$.

b. Find the value of x where g has a minimum and the value of x where g has a maximum. Explain.

5. Let $f(x) = 4 + 27x - x^3$ and let A be the interval $0 \le x \le 3$.

a. Show that $f(x)$ is increasing for x in the interval A.

b. Find the interval B equal to the range (= output) of f when $0 \le x \le 3$.

c. Show that the function $f\colon A \to B$ has an inverse function $g\colon B \to A$.

d. Show that $g(50) = 2$ and find the derivative $g'(50)$.

6. An automobile undergoing emergency braking has an acceleration $a(t) = -3t - 2$ meters/sec^2 at time $t \ge 0$. Its initial velocity $v(0) = v_0$ is not known. You may assume that the initial position $s(0) = 0$ is the origin.

a. Find formulas for the automobile's velocity $v(t)$ and displacement $s(t)$ in terms of the unknown constant v_0.

b. If the automobile travels 84 meters in the first 4 seconds, find v_0.

(continued on the back)

7. Let C_1 be the parabola $y = 12x - 3x^2$ and let C_2 be the cubic curve $y = x^3 - 16x$.

a. Sketch the curves and label their points of intersection for $x \geq 0$. You may use the calculator.

b. Let $\mathcal{R}$ be the region enclosed between C_1 and C_2 for $x \geq 0$. Set up a definite integral, including limits of integration, to represent the AREA of $\mathcal{R}$. Then find the numerical value of this integral. Calculator permitted.

c. Set up (but do not anti-differentiate!) a definite integral, including limits of integration, to represent the VOLUME obtained by rotating $\mathcal{R}$ about the line $x = 5$.

d. Let $\mathcal{S}$ be the region in the FIRST QUADRANT enclosed between the x-**axis** and the parabola C_1. Set up (but do not anti-differentiate!) a definite integral, including limits of integration, to represent the VOLUME obtained by rotating $\mathcal{S}$ about the x-axis.

8. Solve the initial-value problem: $(1 + x^2)\, y \dfrac{dy}{dx} = 4x$ with initial condition $y(0) = 3$.